Energy

ΔTemp

- energy transfered with temp difference is

Heat

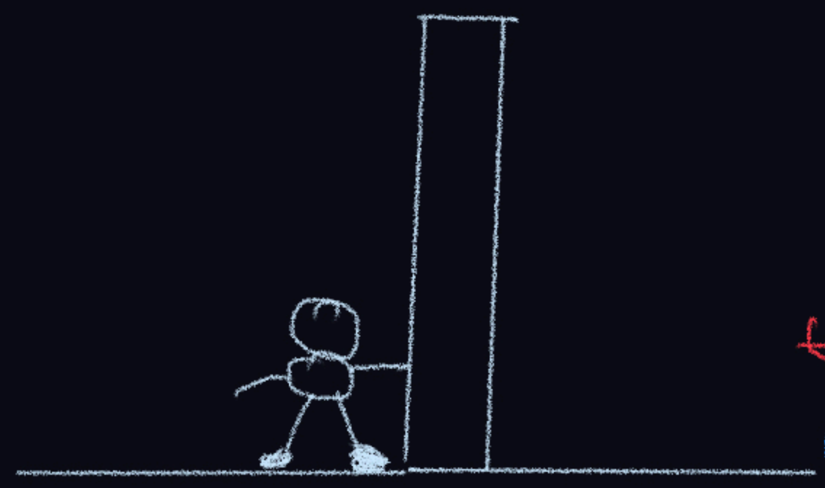
Heat: process of transfer of energy via temperature difference

Force

- energy transferred with the force is

Work

work: process of transfer of energy via force



Force

pipe

You

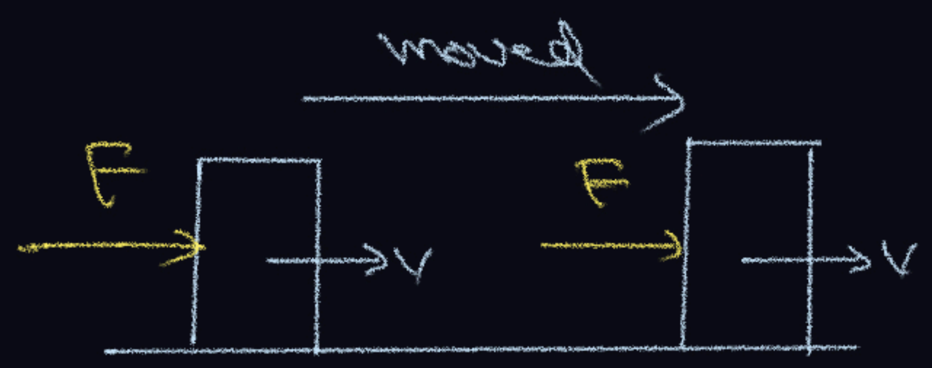
tank

Block

bucket

(force acts like a pipe) to transfer energy

Work done: $W = \sum dW = \sum \vec{F} \cdot d\vec{s}$



$a = \frac{F}{m} = \text{constant}$

$v^2 - u^2 = 2as$

$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mas$

$K_f - K_i = Fs$

$\Delta K = Fs$

$$\boxed{W = Fs}$$

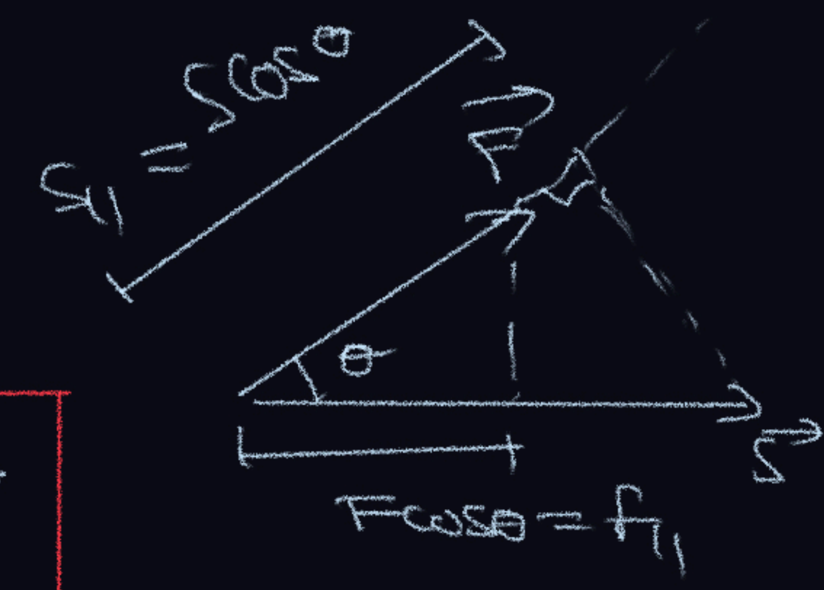
$$\downarrow$$

$W = F s_{\parallel}$

$s_{\parallel}$ = displacement along force

constant force

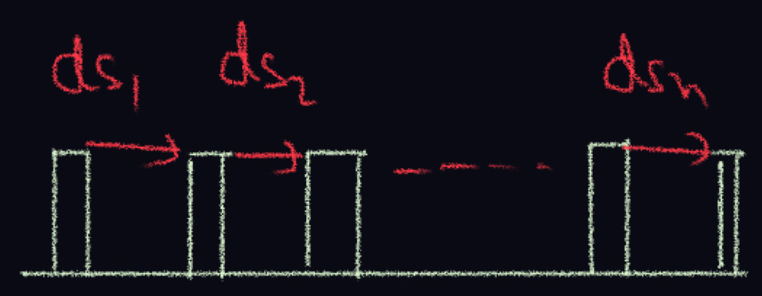
$$\boxed{W = \vec{F} \cdot \vec{s}}$$



$(F\cos\theta)s$ | $F(s\cos\theta)$

$F_{\parallel}s$ | $F s_{\parallel}$

Variable forces:

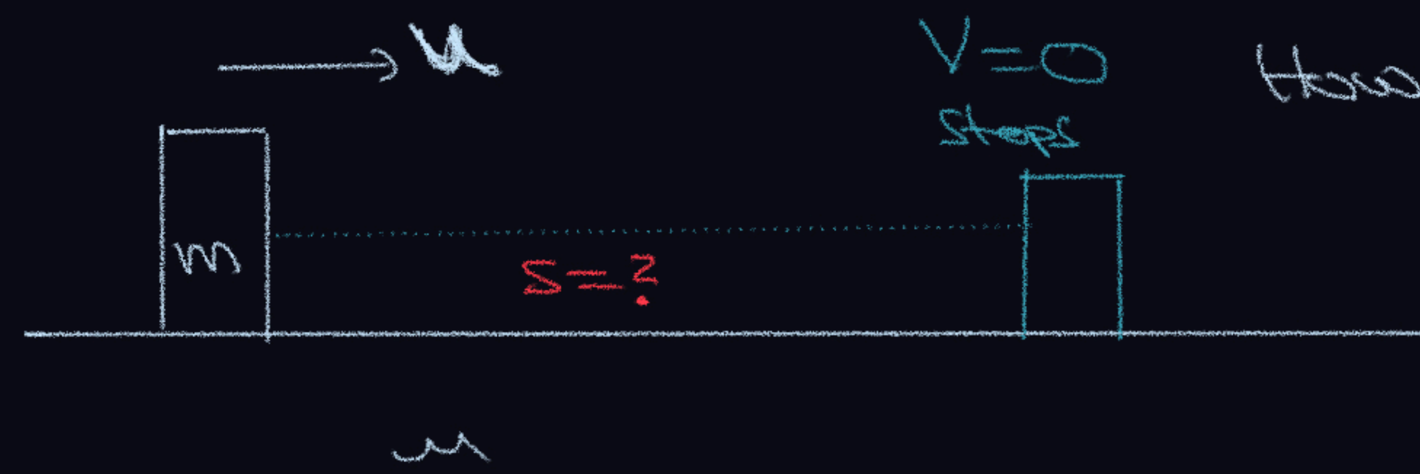


$dW_1 = \vec{F}_1 \cdot d\vec{s}_1$

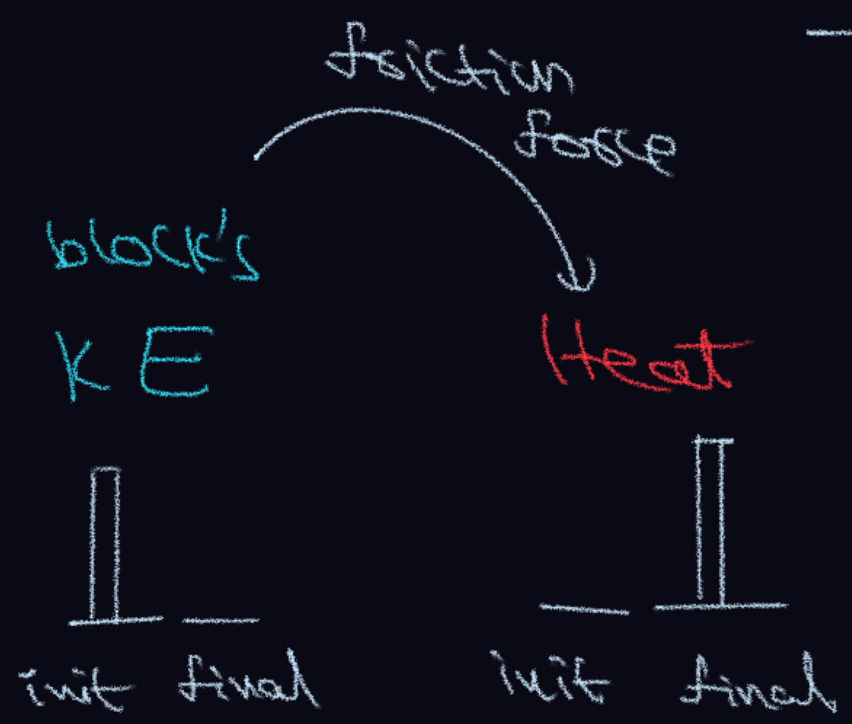
$dW_2 = \vec{F}_2 \cdot d\vec{s}_2$

$$W = \sum_{i=1}^{n} \vec{F}_i \cdot d\vec{s}_i$$

Constant force:

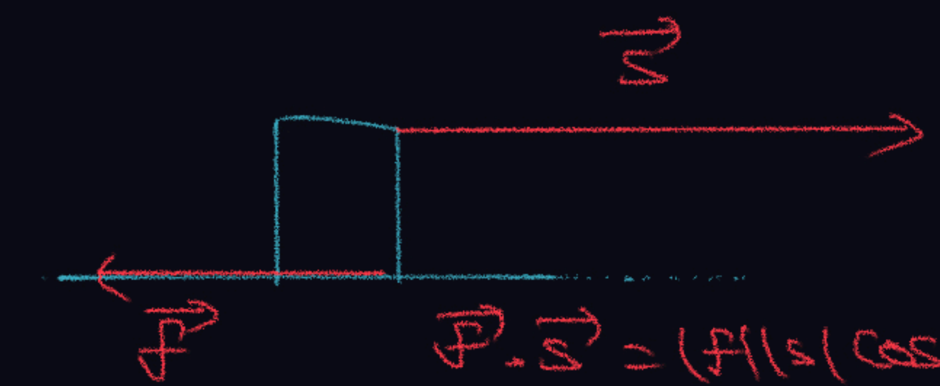


How far does block slide before it stops.



$$f = \text{kinetic} = \mu mg = \text{constant force}$$

$$W = \vec{F}\cdot\vec{s}$$



$$\vec{f}\cdot\vec{s} = |f||s|\cos\pi = -\mu mgs$$

$$W = \mu mg(-\hat{i})\cdot s(\hat{i}) = -\mu mgs \quad (-\hat{i}\cdot\hat{i})$$

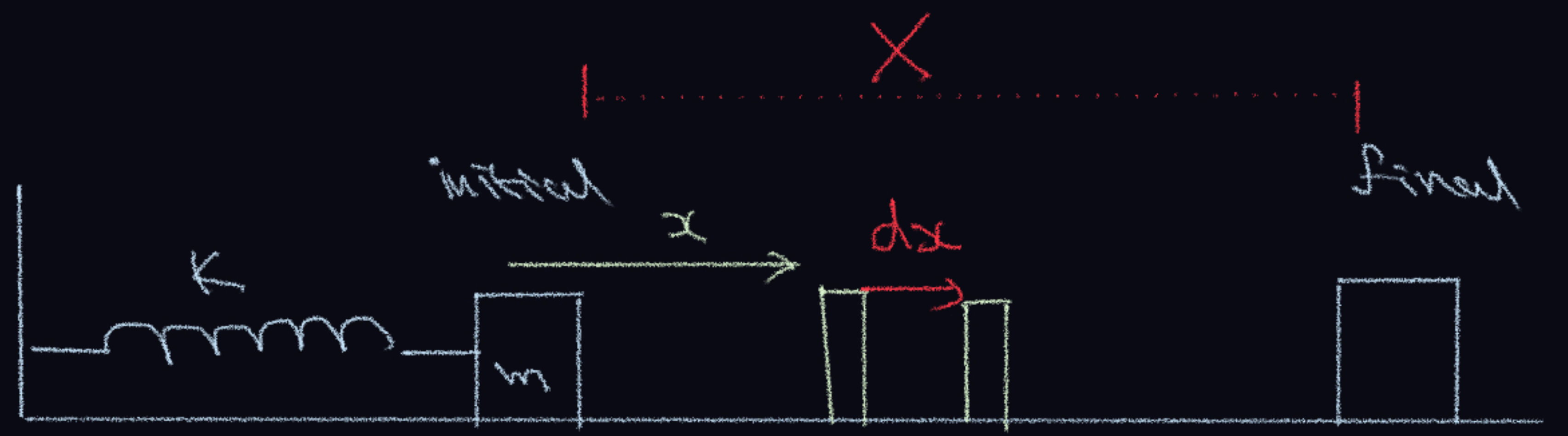
Work on block $= \boxed{W = -\mu mgs}$ = −ve work done by friction

Work energy theorem: $W = \Delta KE$

$$-\mu mgs = 0 - \tfrac{1}{2}mu^2$$

$$s = \frac{u^2}{2\mu g}$$

Variable force:



calculate work done by spring force.

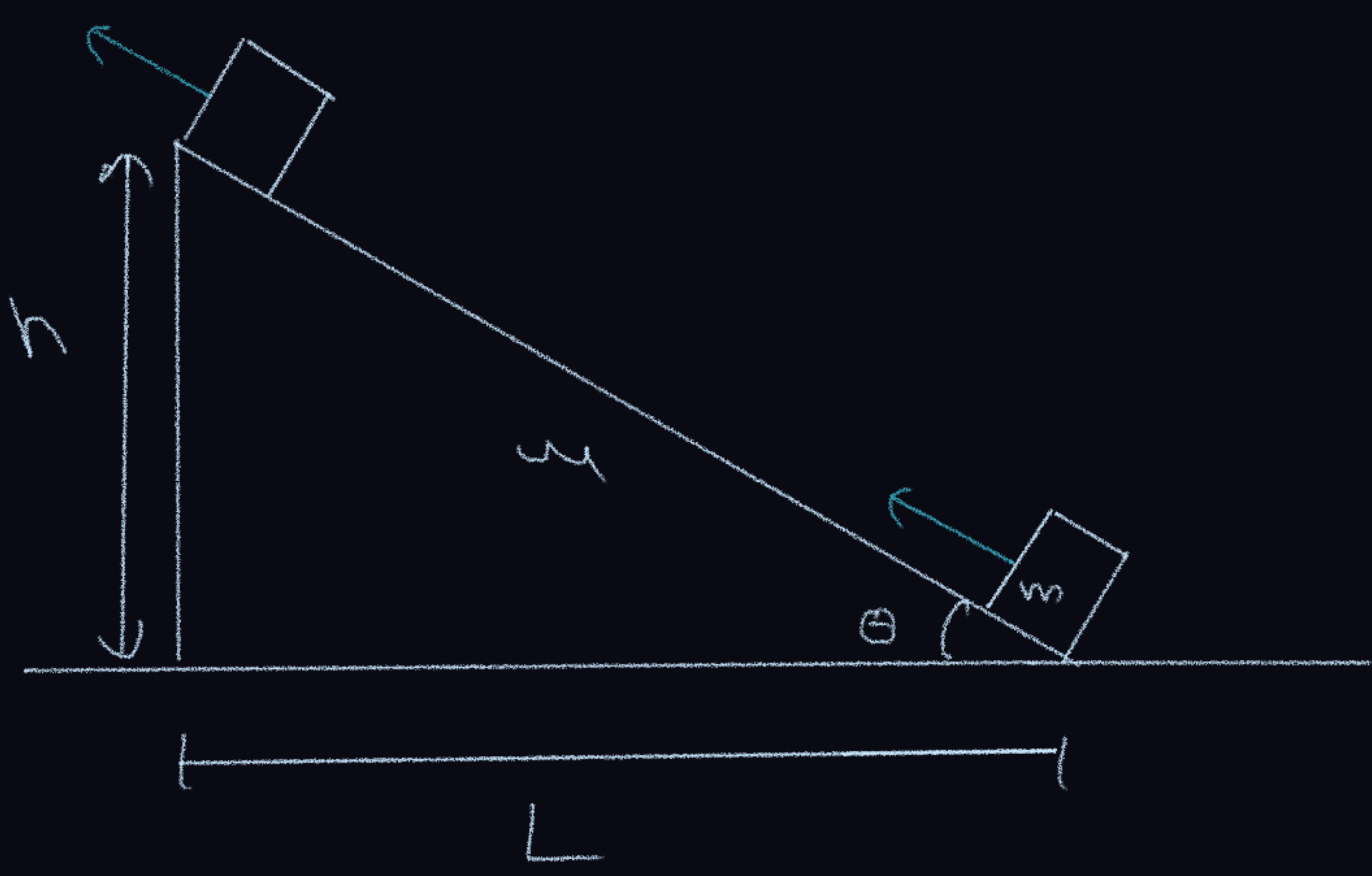
for a given displacement 's'.

- variable force
- general point
- displace ($d\vec{s}$)
- position (x)
- write: dW
- integrate
- eliminate variable
- put limits

$$dW = Kx(-\hat{i}) \cdot dx(\hat{i}) = -Kx\,dx$$

$$W = -K\int x\,dx = -K \left.\frac{x^2}{2}\right|_{0\,=\,\text{initial}}^{l\,=\,\text{final}}$$

$$W = -K\left(\frac{l^2}{2} - \frac{0^2}{2}\right) = -\frac{Kl^2}{2}$$

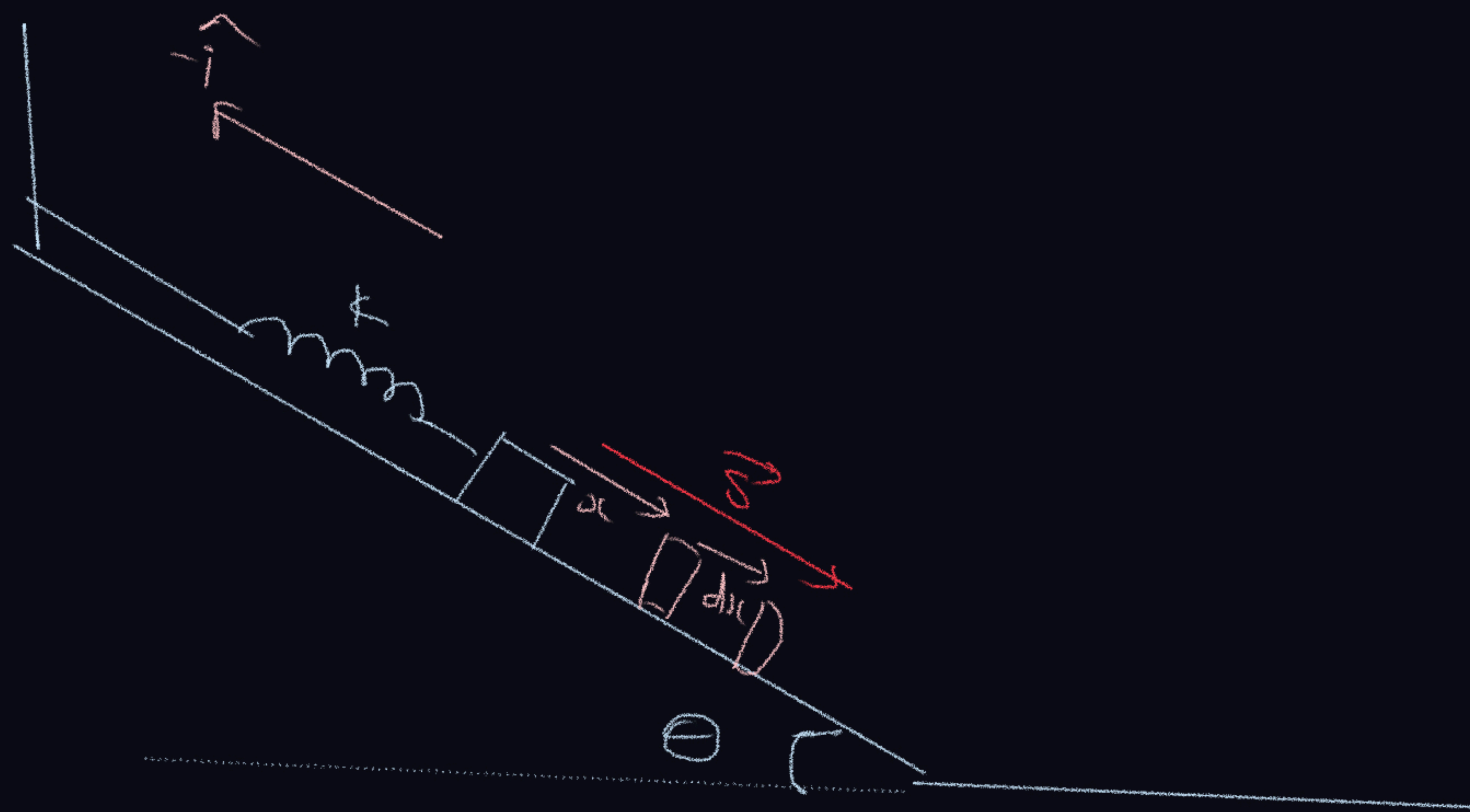


Find work done by friction force.

$$W = -\mu m g\, h\cot\theta = -\mu m g L$$

$$\cot\theta = \frac{L}{h}$$

$$h\cot\theta = L$$

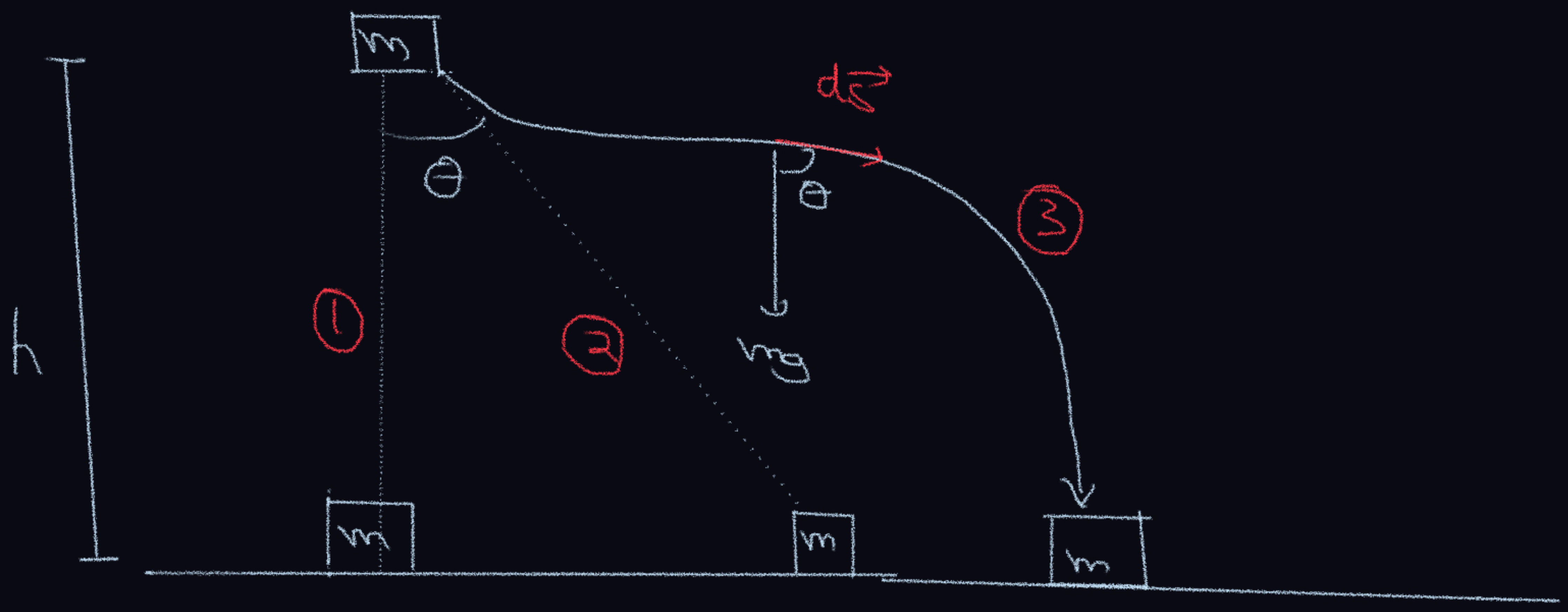


$$dW = kx(-\hat{i}) \cdot dx(\hat{i})$$

$$dW = -kx\,dx$$

$$W = \int -kx\,dx = -k\frac{x^2}{2}\Big|_0^{\delta}$$

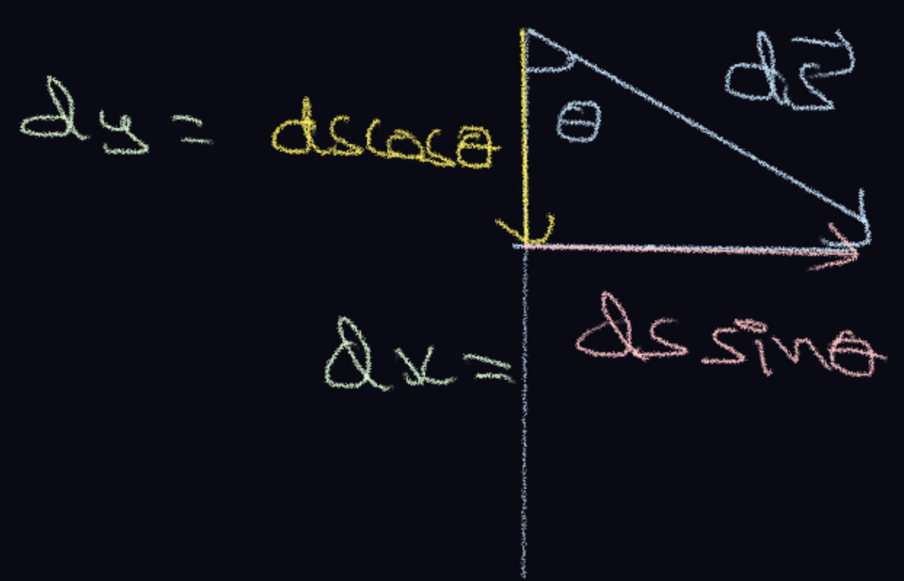
$$W = -k\frac{\delta^2}{2}$$

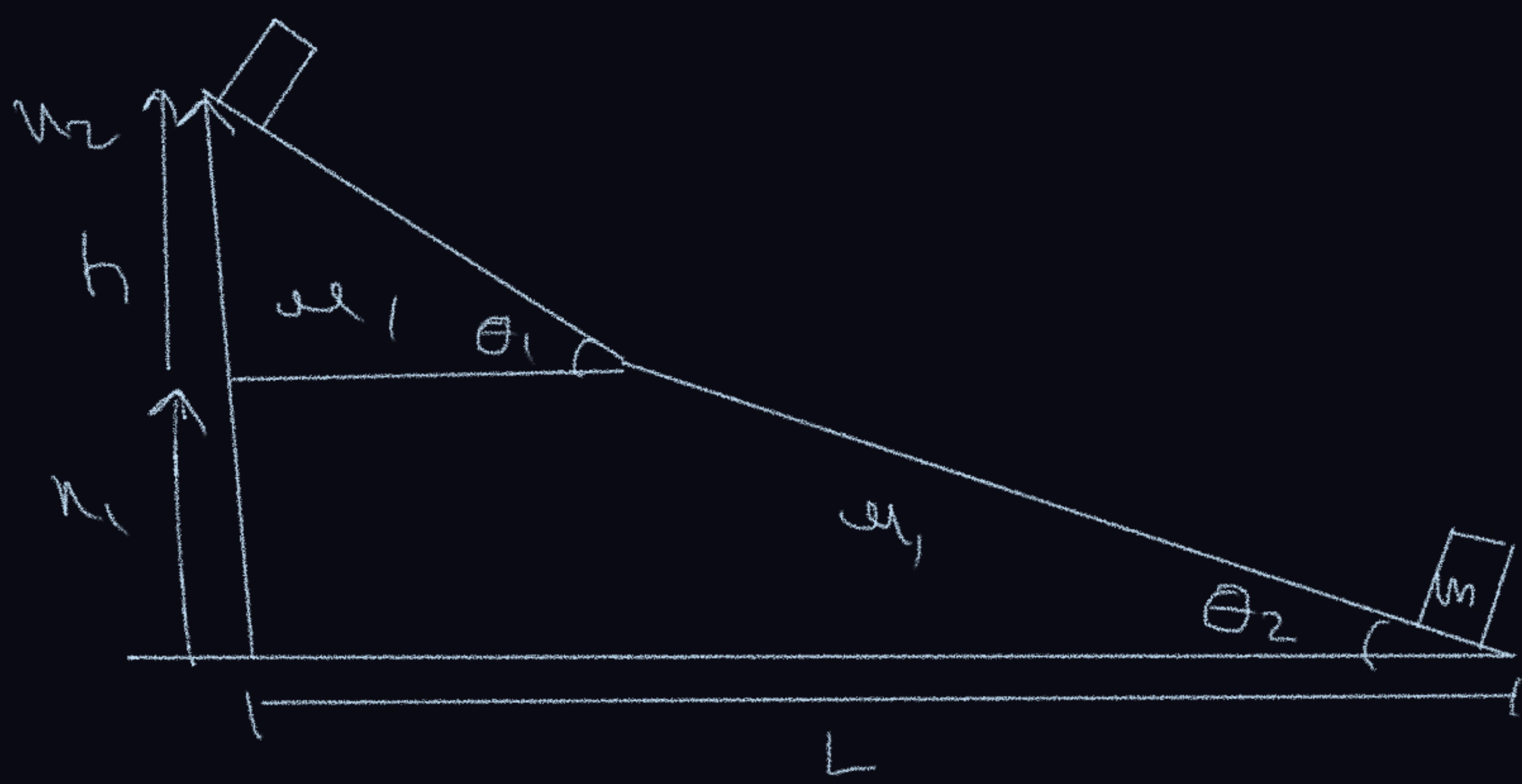


$$dW = \vec{F} \cdot d\vec{s} = F\,ds\cos\theta$$

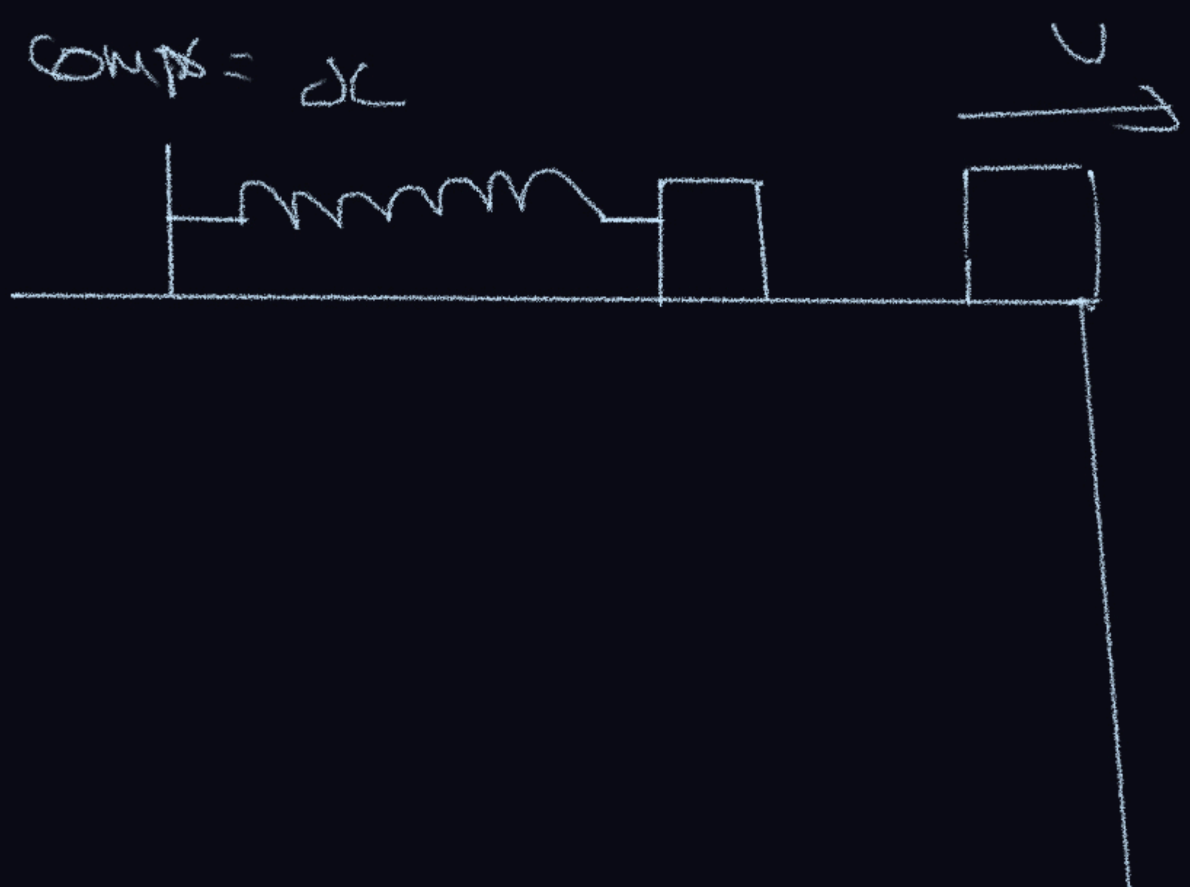
$$dW = F\,dy = mg\,dy$$

$$W = mg\int dy = mgh$$

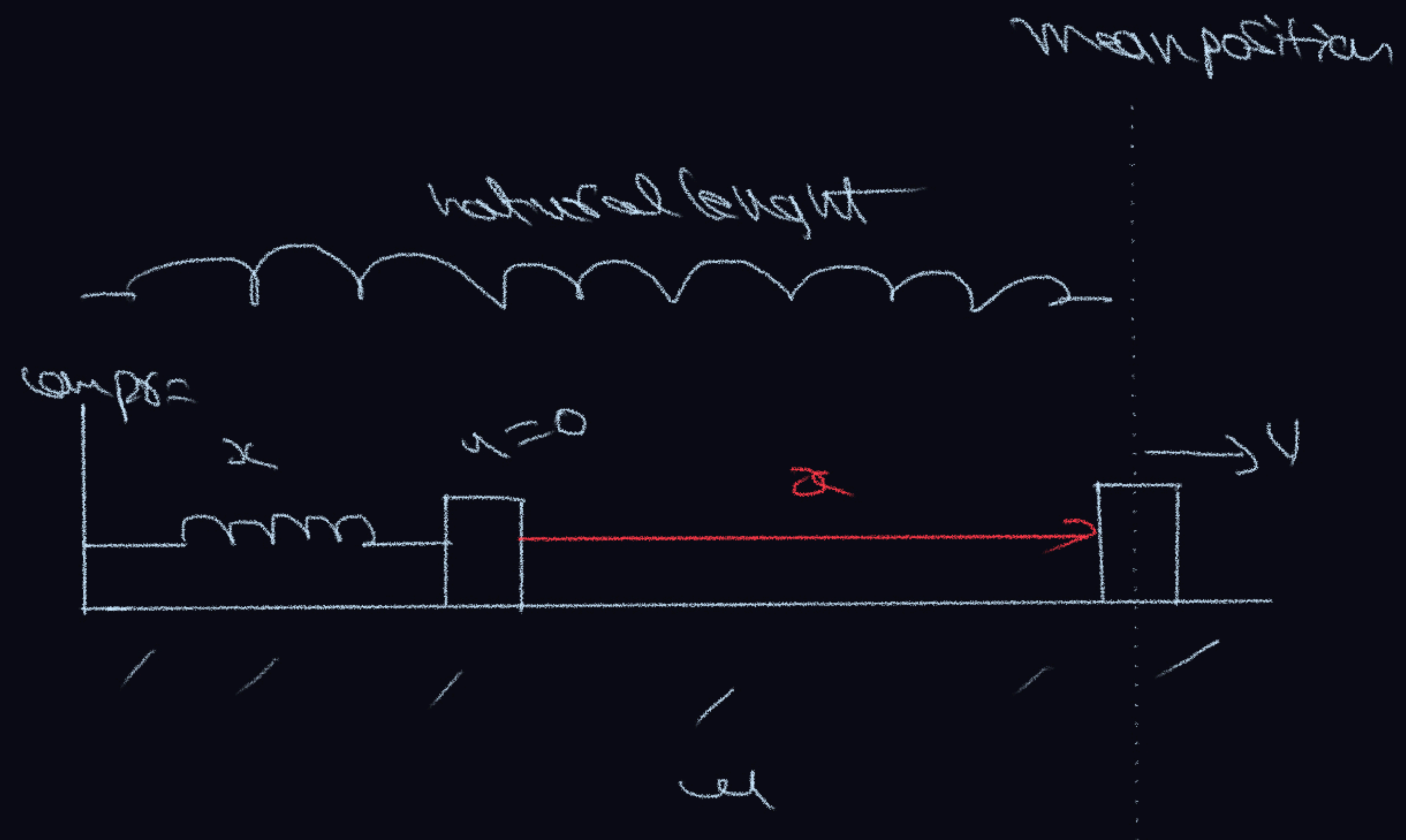




$$W = -\mu_1 m g L$$

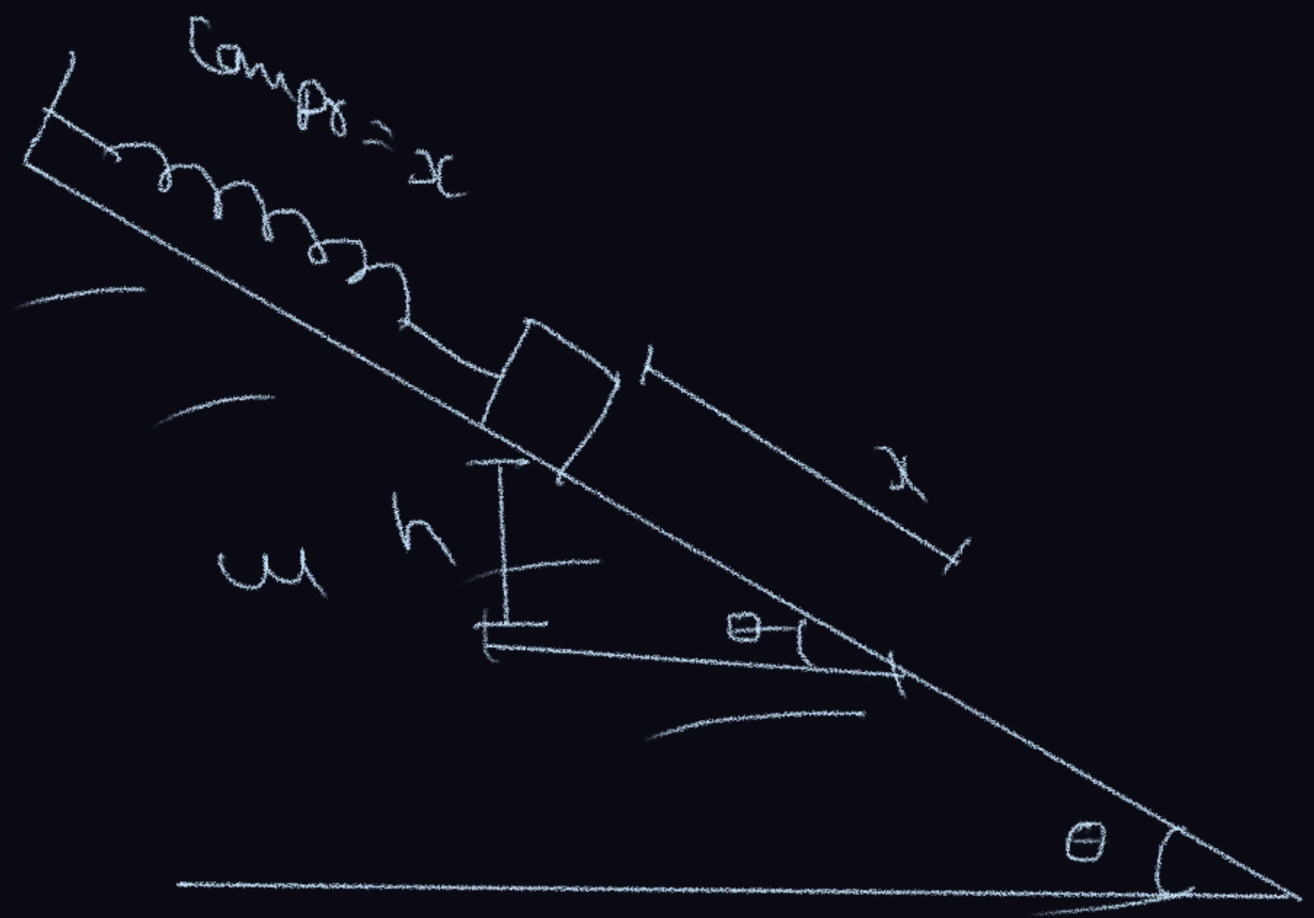


$W_{spr} = \Delta KE$



$$\frac{1}{2}kx^2 - \mu mgx = \frac{1}{2}mv^2$$

Find velocity when block reaches mean position

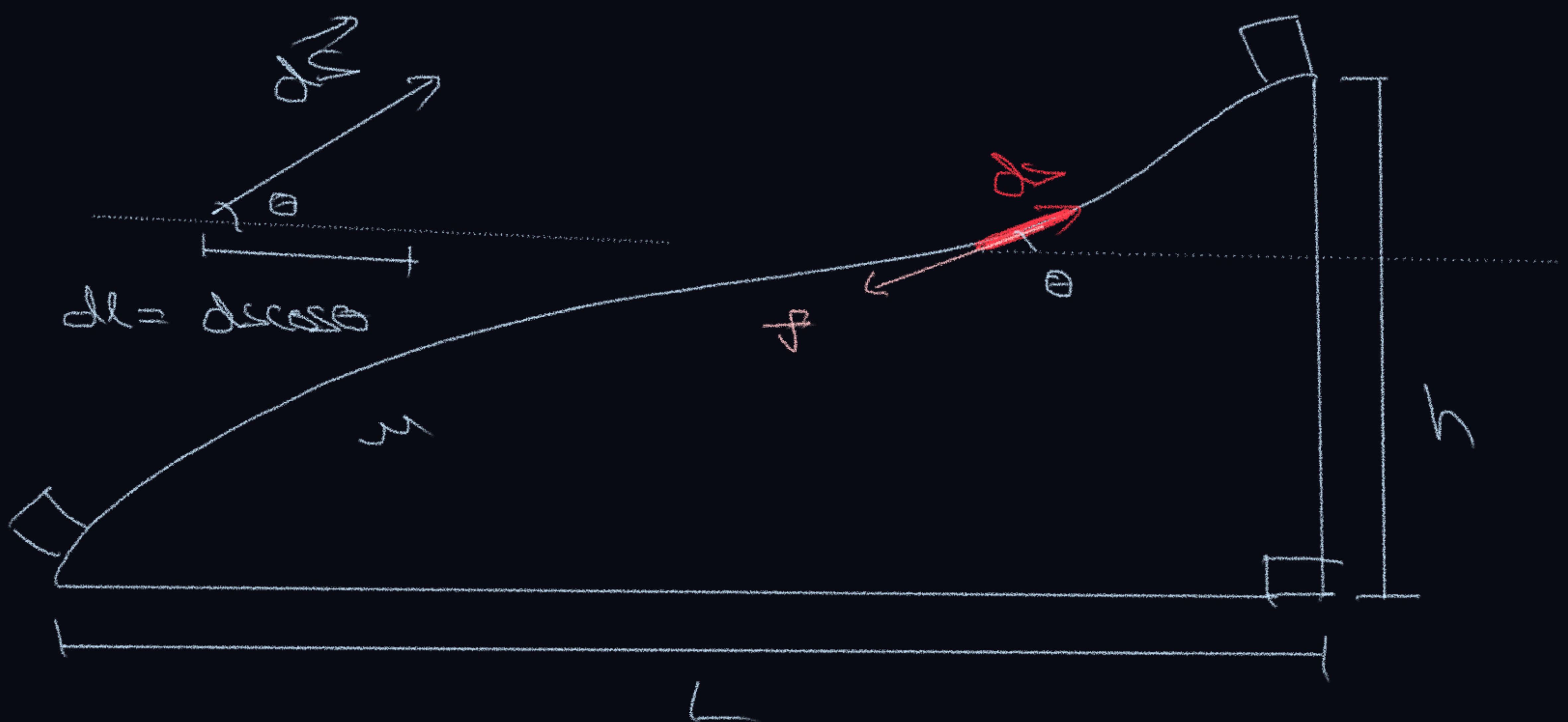


$$W = -\mu mg\cos\theta\, x$$

$$\frac{1}{2}kx^2 - \mu mg\, x\cos\theta + mg\, x\sin\theta$$

$$= \frac{1}{2}mv^2 - 0$$

work by friction?

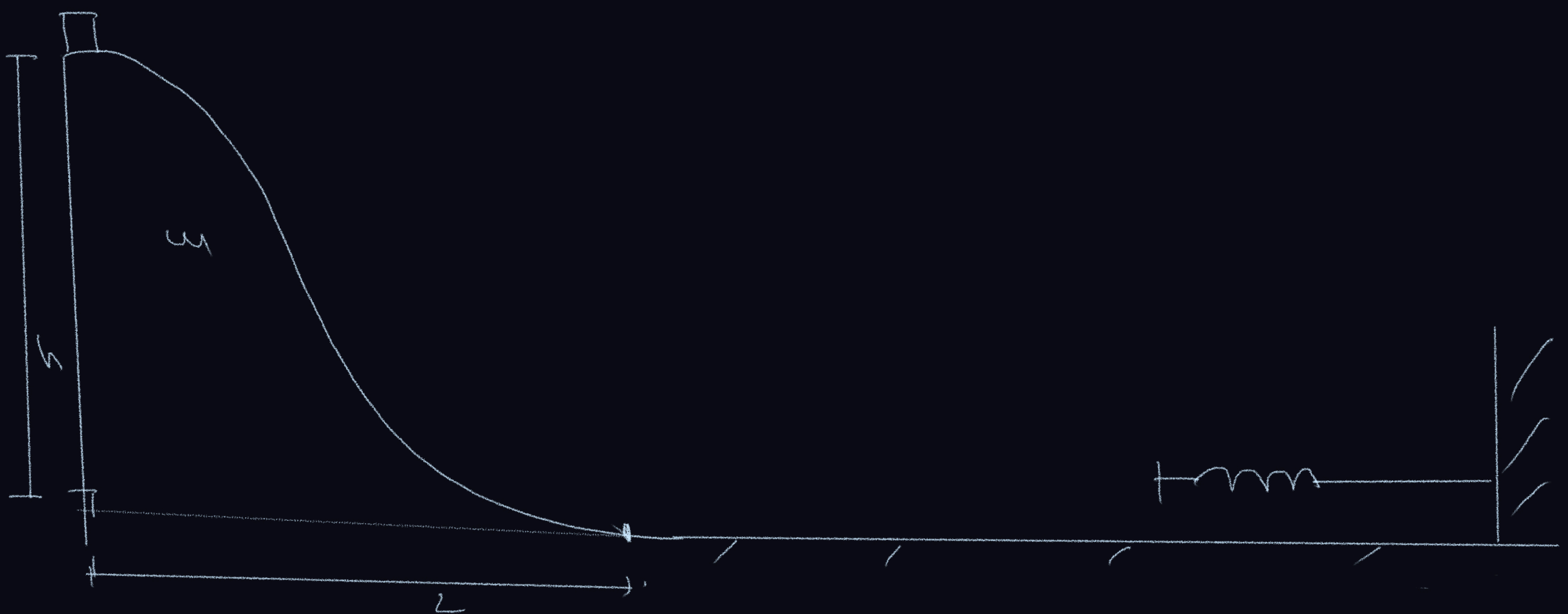


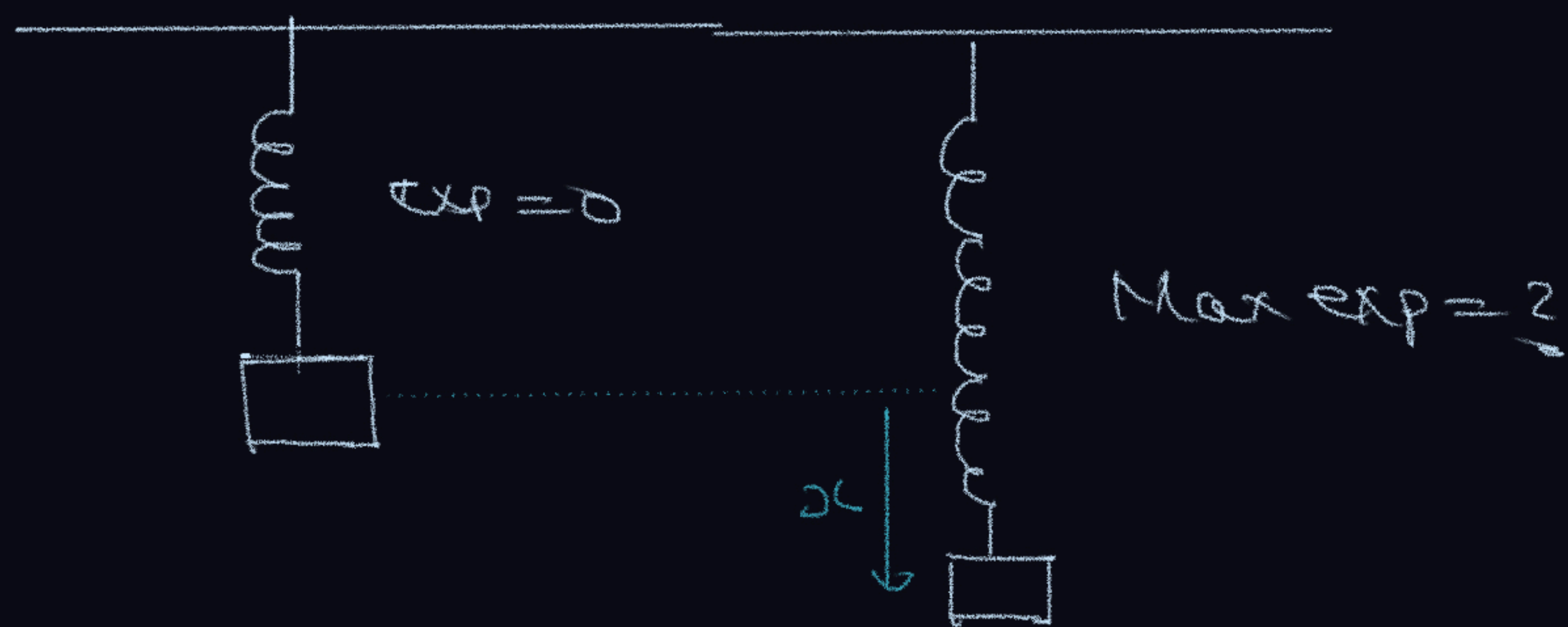
general point

$$dW = \vec{f}\cdot d\vec{s} = -f\,ds = -\mu mg\cos\theta\,ds = -\mu mg\,(ds\cos\theta)$$

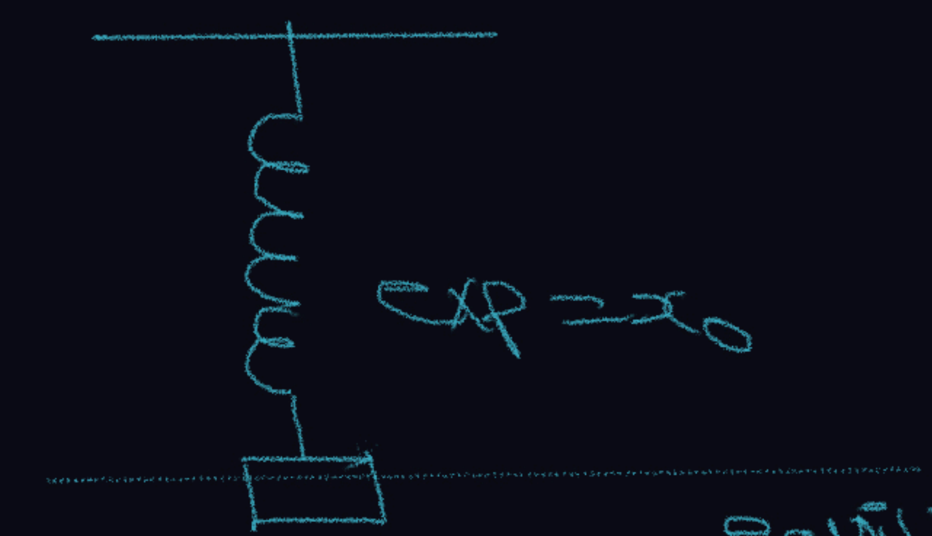
$$dW = -\mu mg\,dl \qquad W = -\mu mg\int dl = -\mu mgL$$

Wall = AKF





where is zero net force?



$$x_0 = \frac{mg}{k}$$

find max expansion?

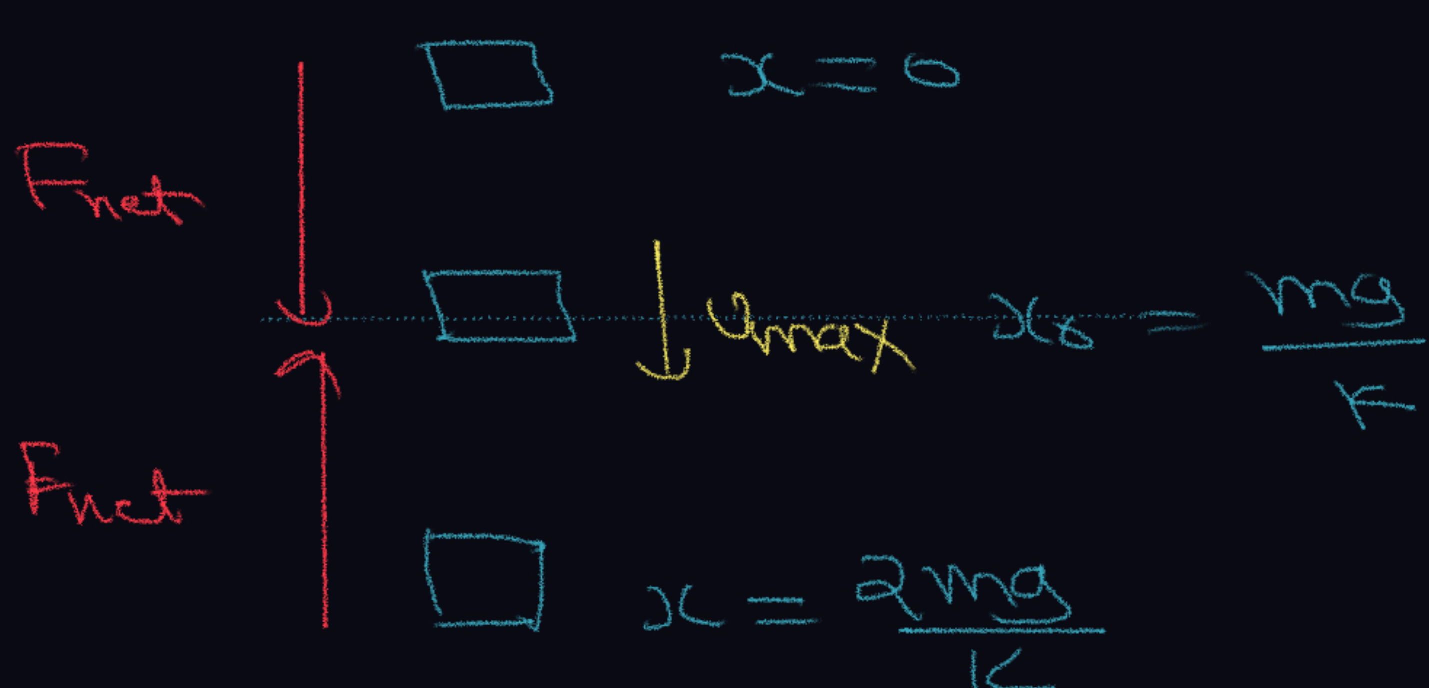
Max exp = x

System: block

$$W_g + W_s = \Delta KE$$

$$-\frac{1}{2}kx^2 + mgx = 0 \qquad x_{max} = \frac{2mg}{K}$$

what is max velocity it can acheive?

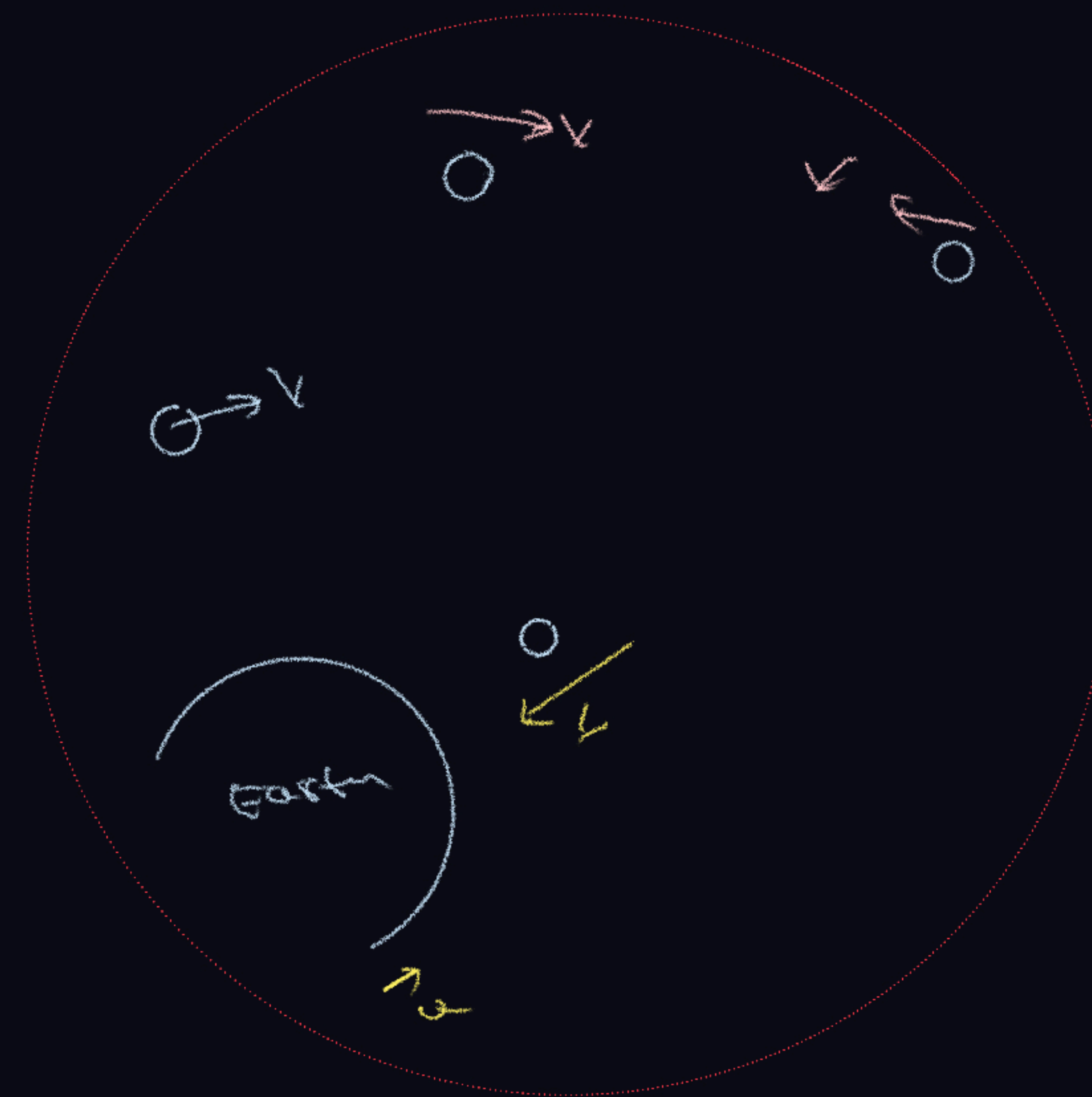


$$mgx_0 - \frac{1}{2}kx_0^2 = \frac{1}{2}mV_m^2 - 0$$

$$V_m^2 = 2g\frac{mg}{k} - \frac{k}{m}\frac{(mg)^2}{k^2} = \frac{mg}{k}(2g - g)$$

$$V_m = \sqrt{\frac{m}{k}}\,g$$

System of particles:



$\text{You} \xrightarrow{\text{Force}} \text{ball}$

water ═══ water

pipe

You = 100J ⟶ 0J

$W_F = 20J$

You = 80J ⟶ 20J

$W_F = 20J$

$E_f - E_i = -20J$

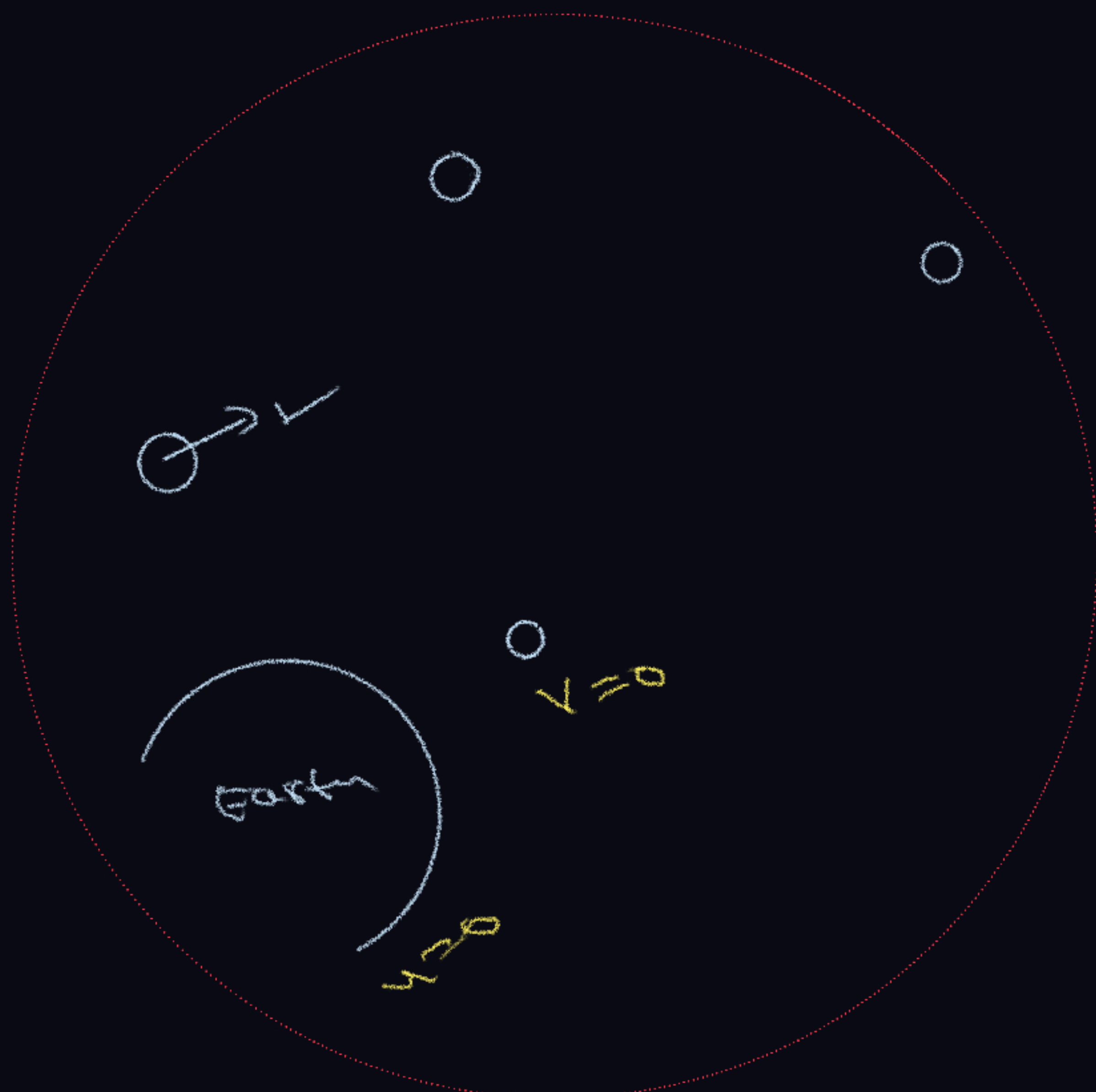
$W_F = -\Delta E$

$\text{P.E} \xrightarrow{W_{\text{gravitat}}} \text{ball} + \text{Earth}$

energy associated with the configuration/arrangement of system of particles

$$\boxed{W_g = -\Delta PE}$$

System of particles:



$W_{all} = \Delta KE$

$W_{ext} + W_{non} + W_{cons} = \Delta K$

$W_{cons} = -\Delta PE$

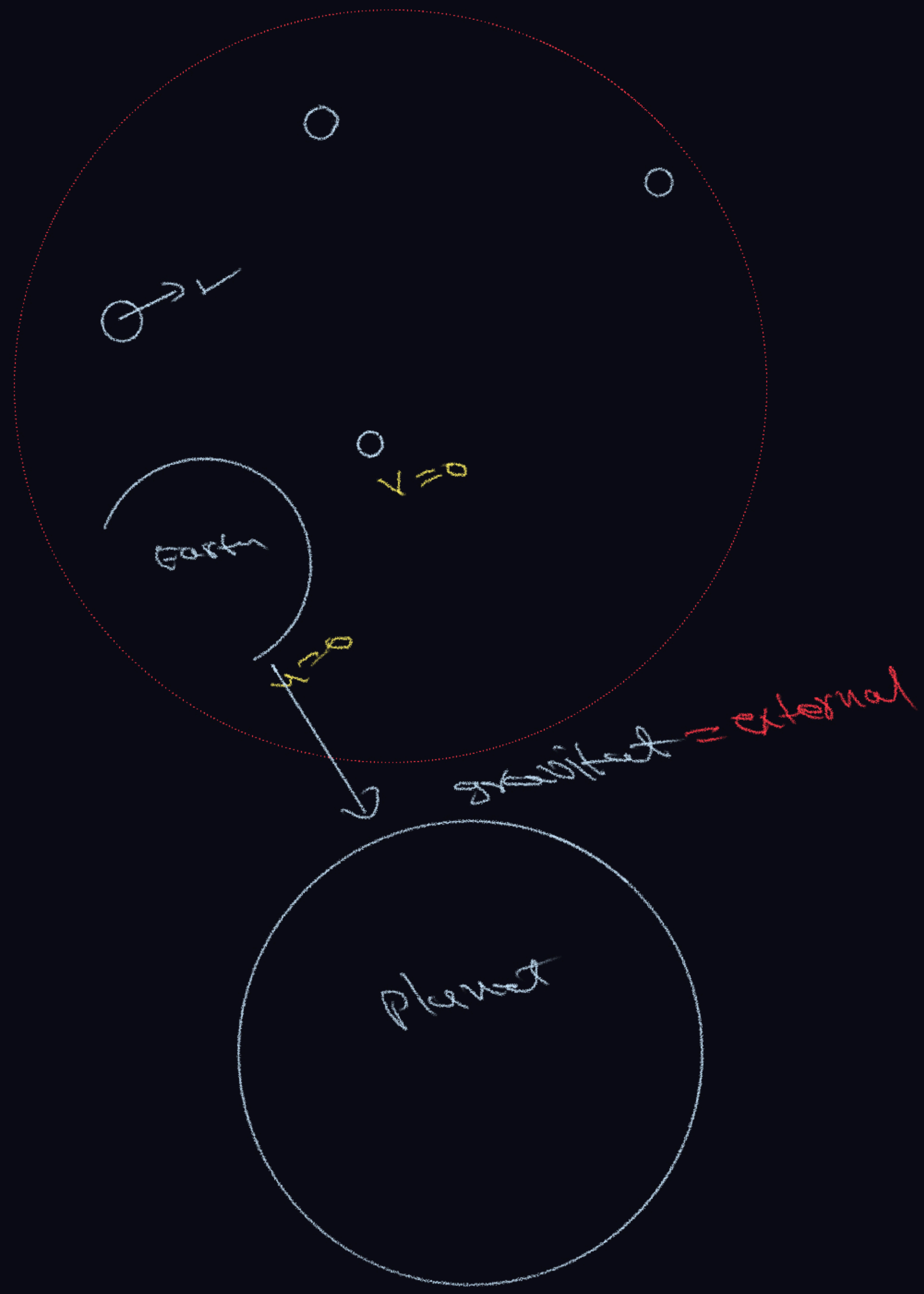
$W_{ext} + W_{non} = \Delta K + \Delta P$

P.E $\xrightarrow{W_{gravitat}}$ ball + Earth

energy associated with the configuration/arrangement of system of particles

$$\boxed{W_g = -\Delta PE}$$

System of particles:



conservative

PE $\rightleftarrows$ KE

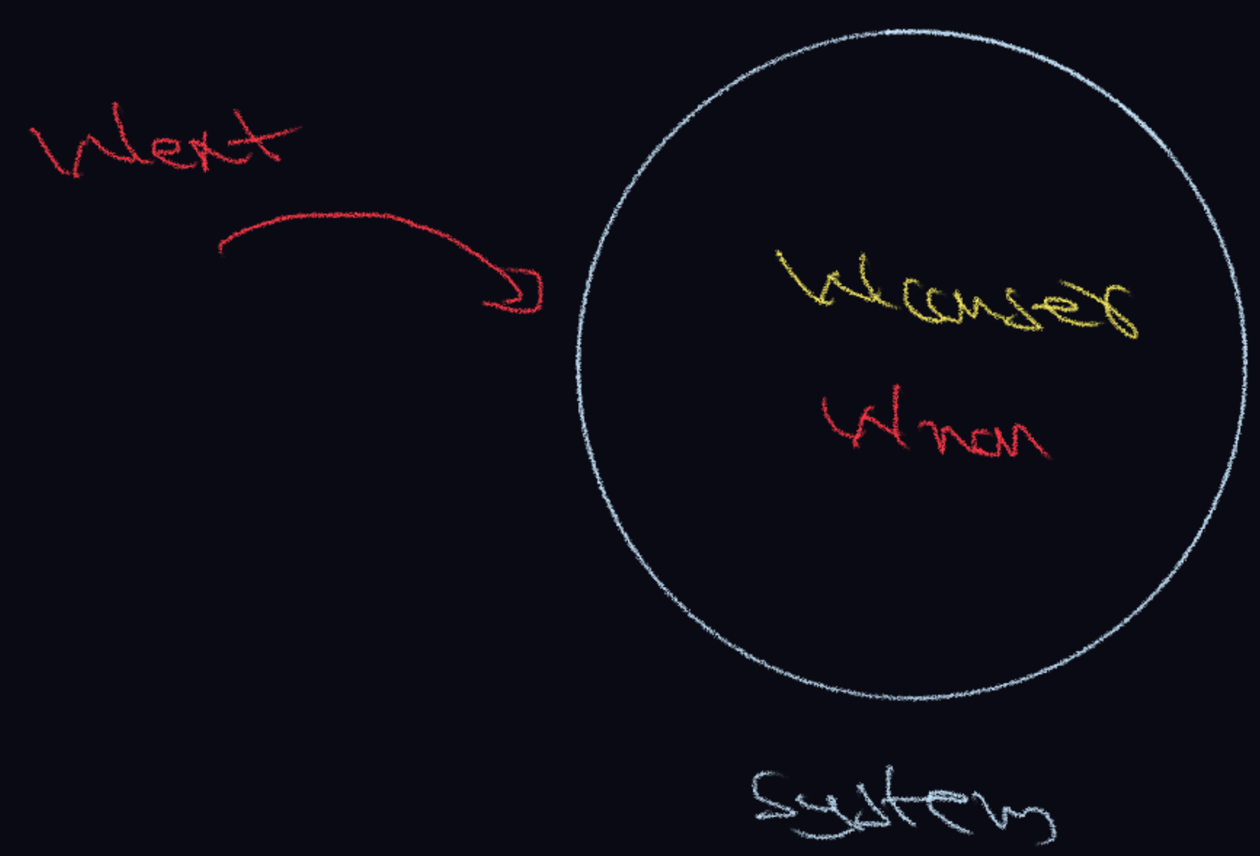
friction

KE $\rightarrow$ Heat

PE $\rightarrow$ Heat

external force

KE + PE $\rightleftarrows$ outside



$$\Delta K + \Delta PE = W_{ext} + W_{non}$$

Forces

External

K + P ⇄ out

Internal

Nonconserv

K ⇄ Heat (out)

Conservative

K ⇄ P

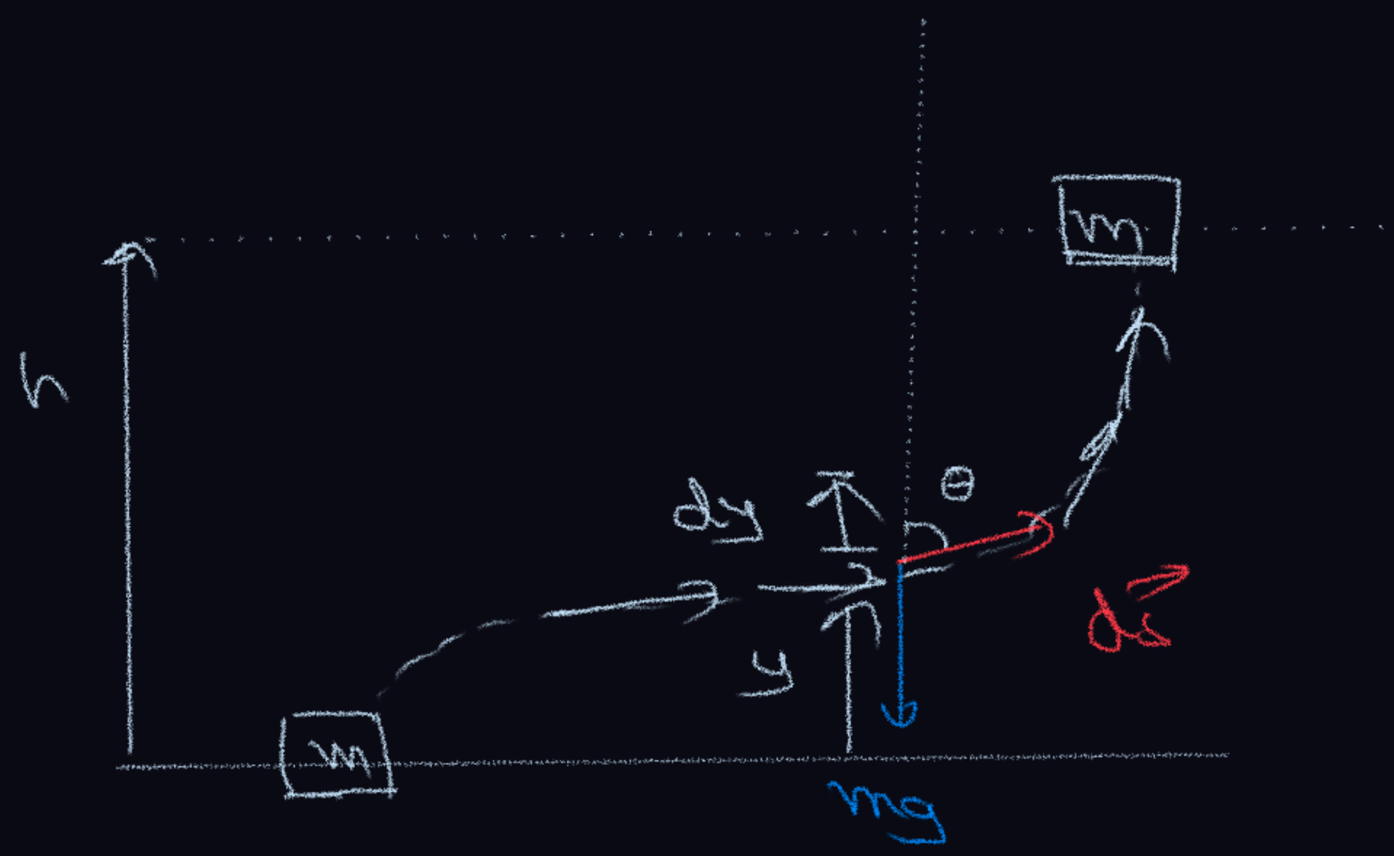
$$\boxed{W_{ext} + W_{ncn} = \Delta K + \Delta P}$$

$$W_{all} = \Delta K$$

$$W_{all} = W_{ext} + W_{ncn} + W_{cons}$$

gravitational Potential:

$W_g = -\Delta U$



$dW = \vec{F} \cdot d\vec{s} = mg\, ds \cos(\pi - \theta)$

$dW = -mg\, ds \cos\theta$

$W = -mg \int ds \cos\theta = -mg \int dy$

$W = -mgy \Big|_0^h = -mgh$

$-(U_f - U_i) = W_g$

$U_i - U_f = W_g$

$U_i = U_f + W_g$

$U_i = U_f - mgh$

zero potial level

$\longrightarrow U_f = U$

$\longrightarrow U_i = 0$

$0 = U - mgh$

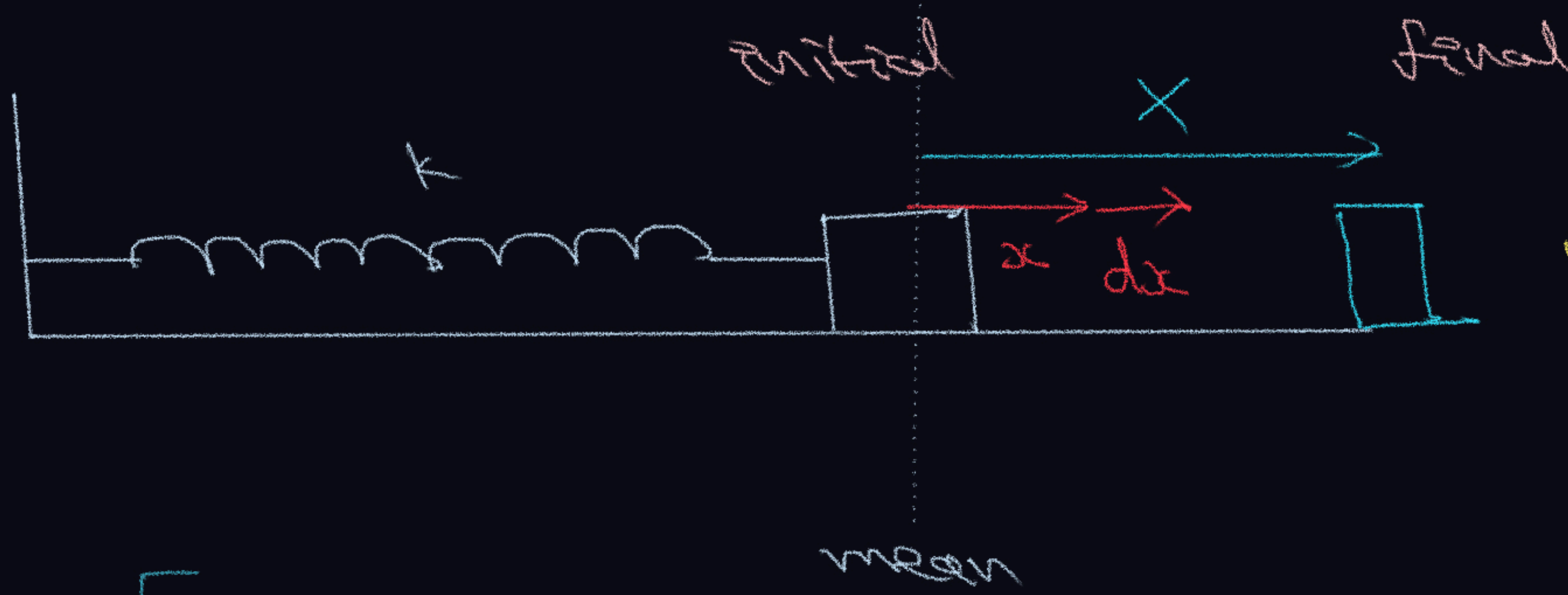
$U = mgh$

$\longrightarrow U_f = 0$

$\longrightarrow U_i = U$

$U = -mgh$

Spring potential energy:



$$W_{spring} = -\Delta U$$

$$dW = \vec{F}\cdot d\vec{s} = kx(-\hat{i})\cdot dx(\hat{i}) = -kx\,dx$$

$$W = \int -kx\,dx = -k\frac{x^2}{2}\Big|_0^X = -k\frac{X^2}{2}$$

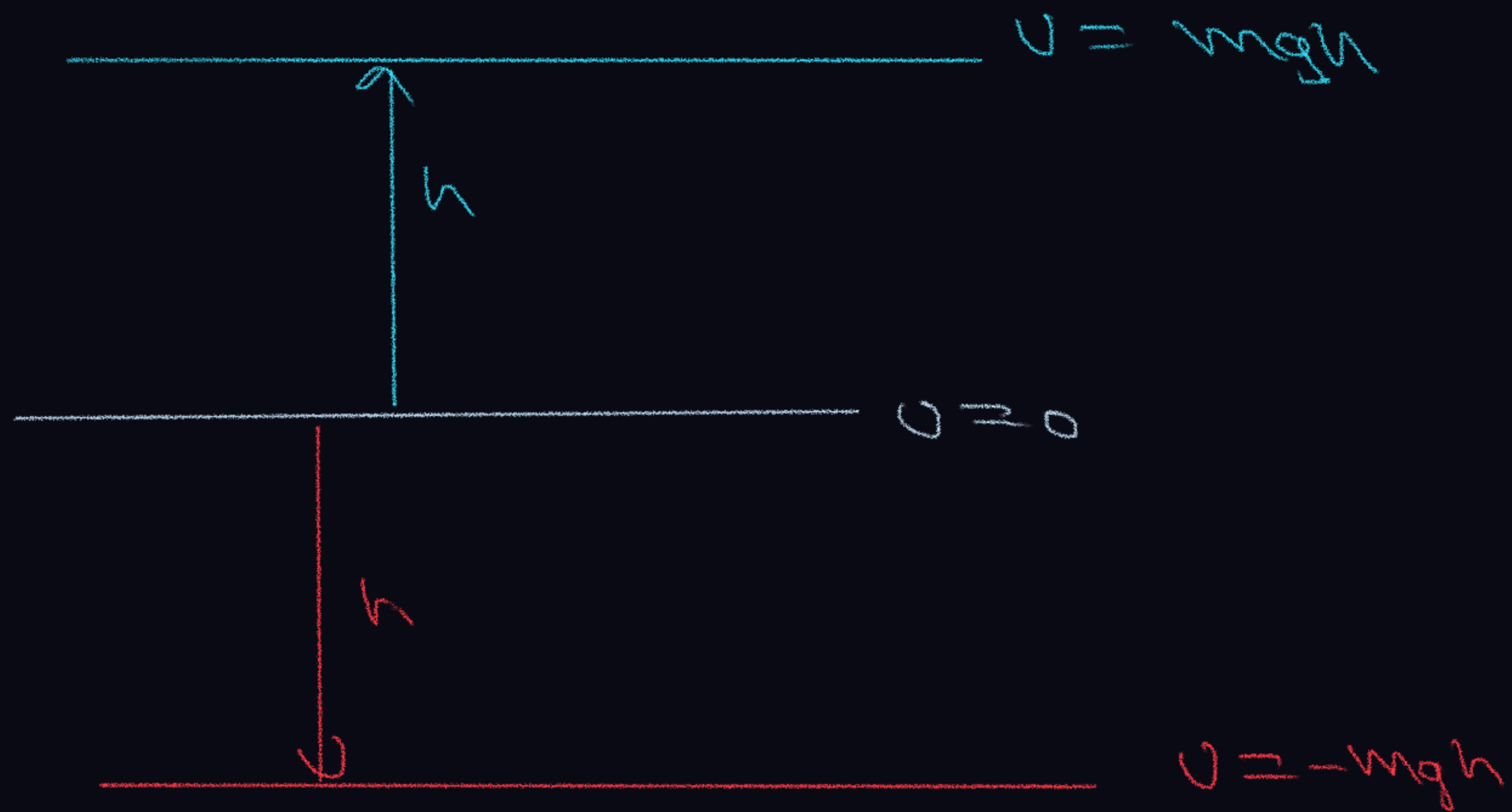
$$\boxed{\therefore W_s = -\frac{1}{2}kX^2}$$

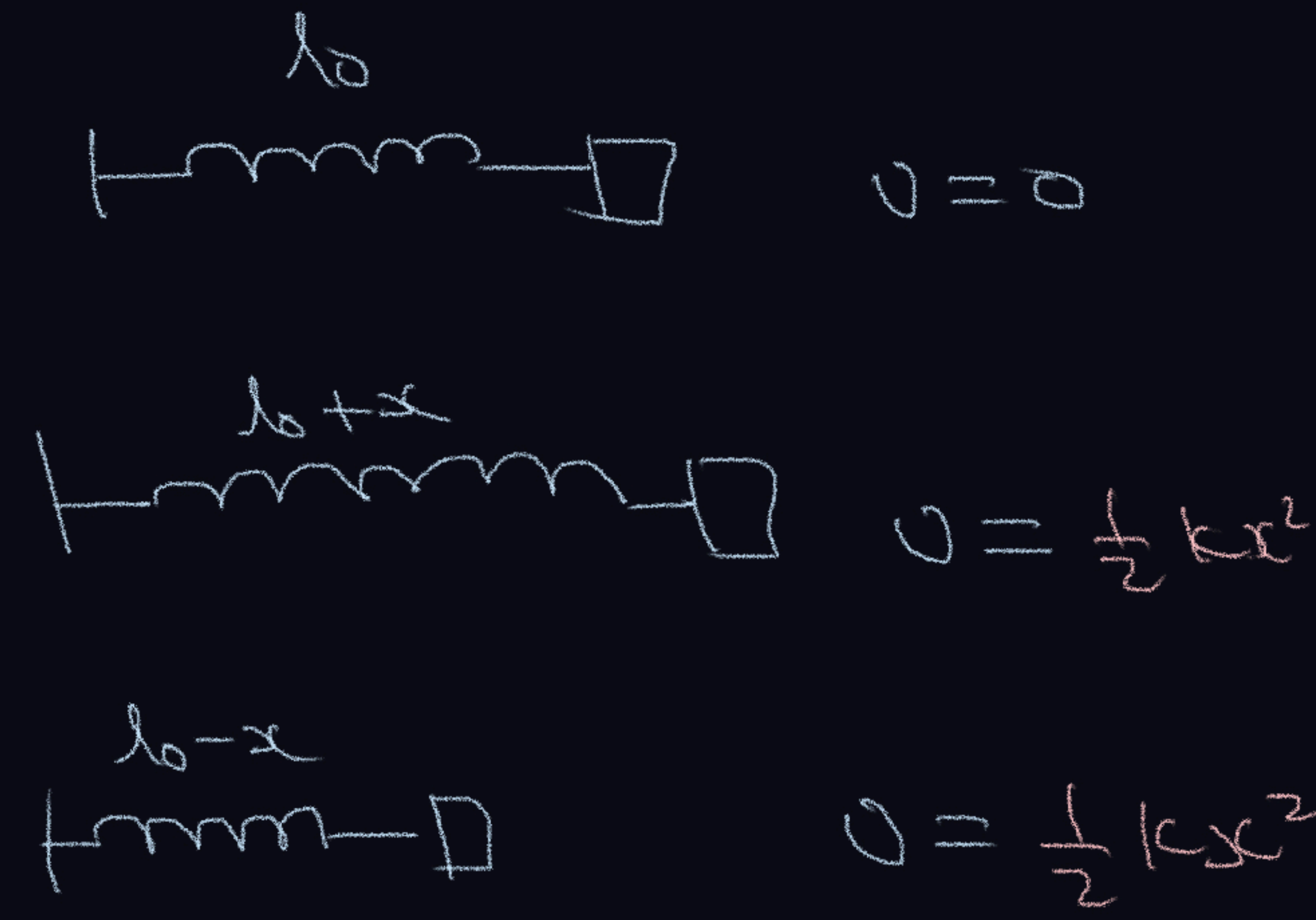
$$-\frac{1}{2}kx^2 = -(U_f - U_i)$$

$$\boxed{\therefore U_f = U_i + \frac{1}{2}kx^2}$$

If $U_{mean} = 0$

$$U = \frac{1}{2}kx^2$$





$W_s = -ve = -\frac{1}{2}kx^2$

Spring PE $\longrightarrow$ block

$\underbrace{W_s}_{-ve} = -\Delta U$

$\therefore \Delta U > 0$

$\boxed{U > 0}$

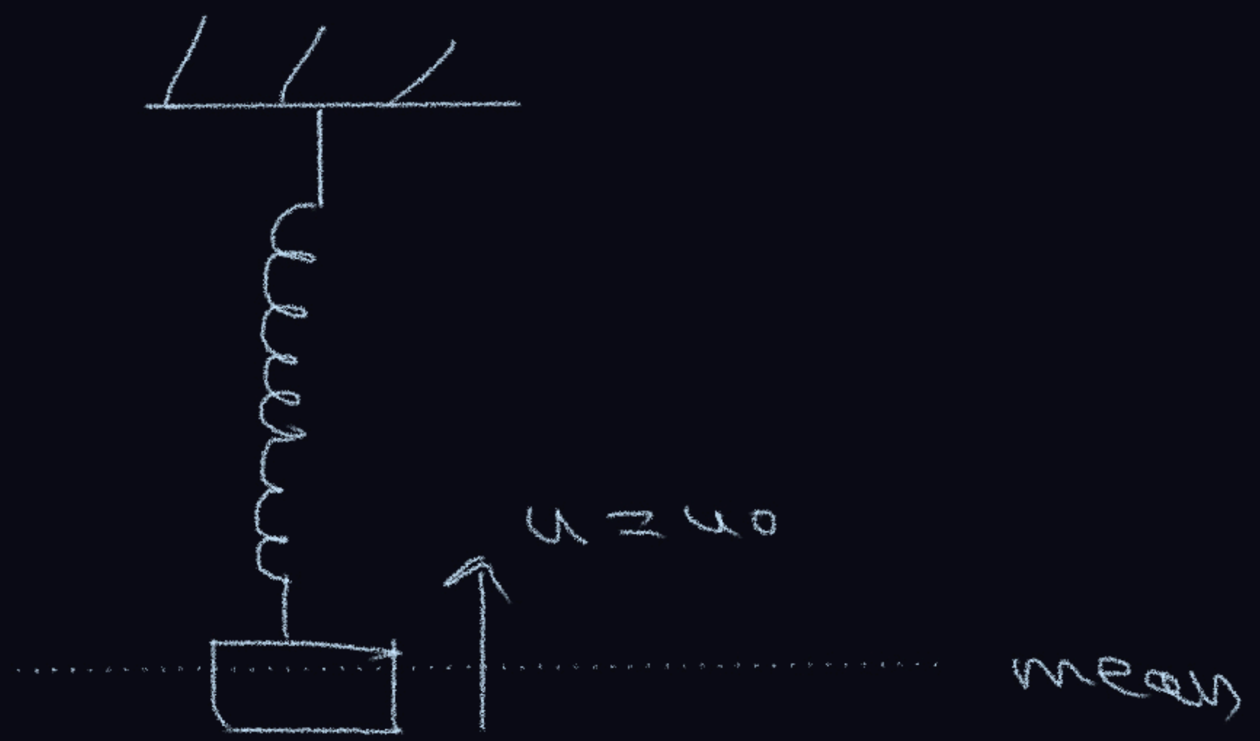
$U - 0 > 0$

mean

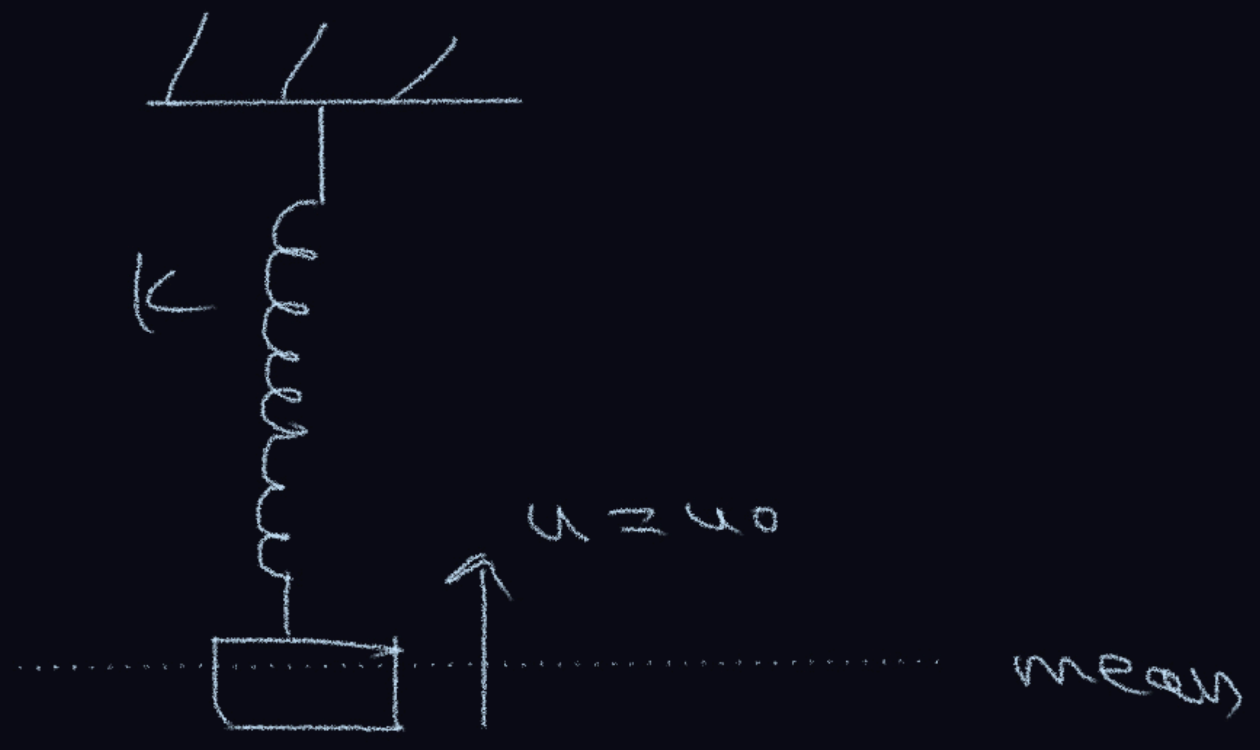
Conservation Mechanical energy:

IF $W_{ext} = 0$ $W_{nc} = 0$ $\Rightarrow$ $\Delta K + \Delta P =$

$$\boxed{K_1 + P_1 = K_2 + P_2}$$



Find Max compression

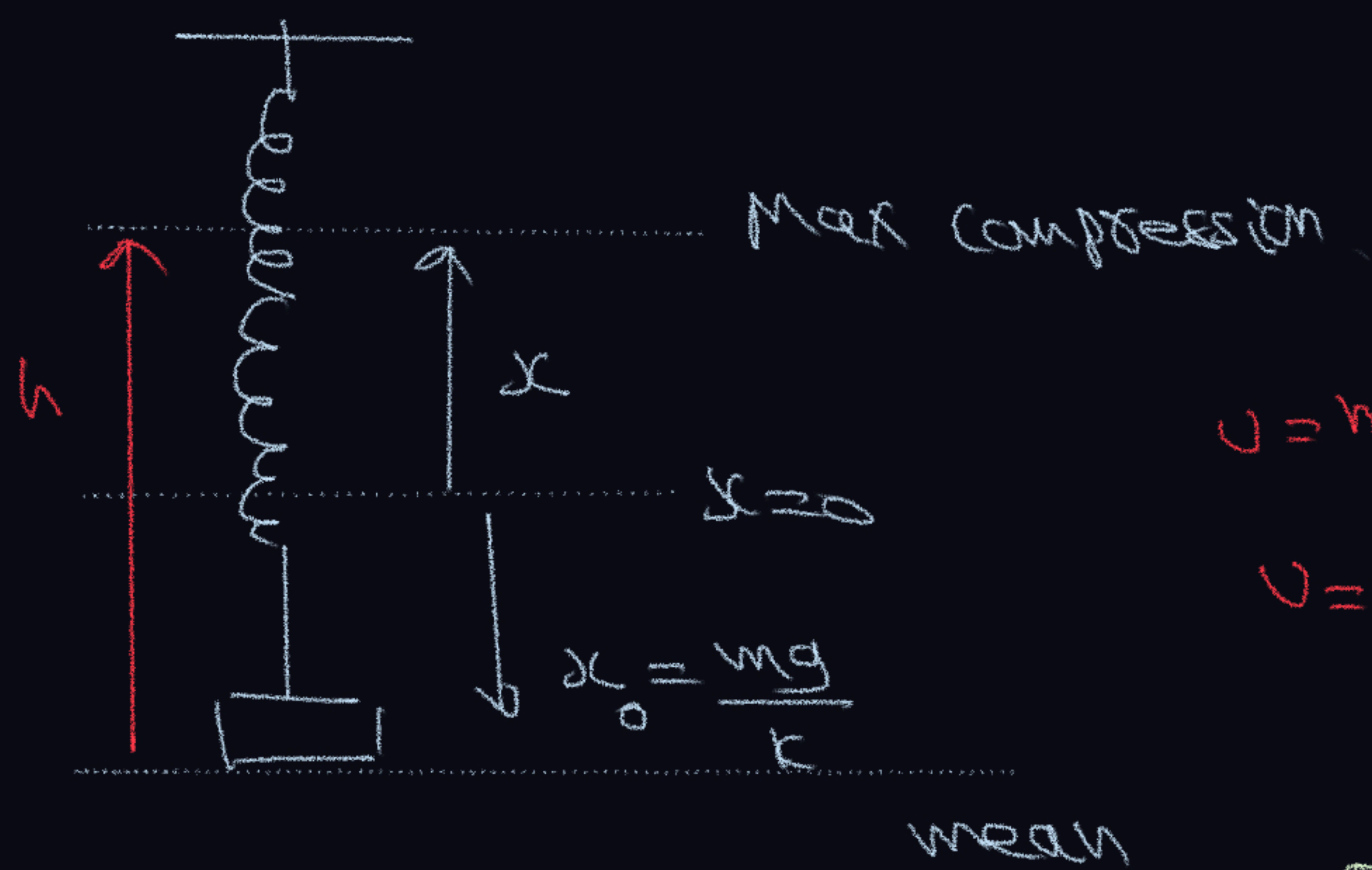


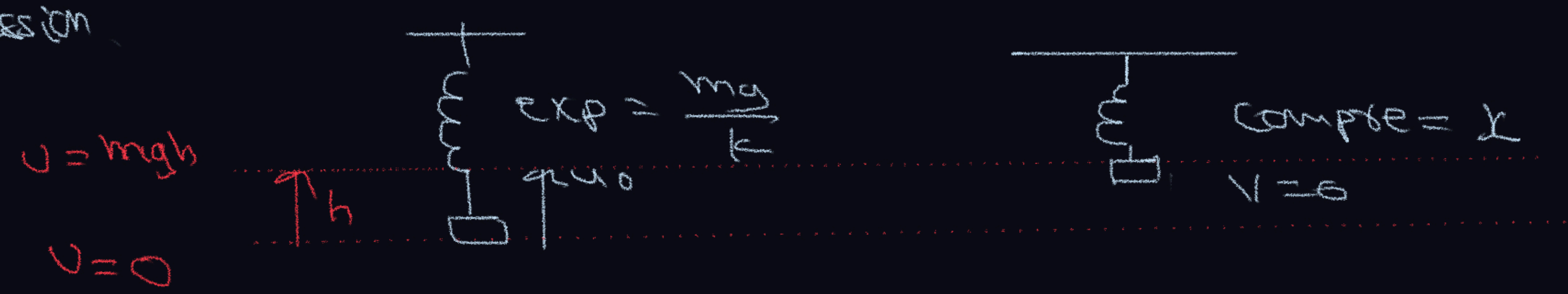
Find Max compression

System: block+spring+Earth

$W_{ext} = 0 \qquad W_{non} = 0$

$$K_1 + P_1 = K_2 + P_2$$

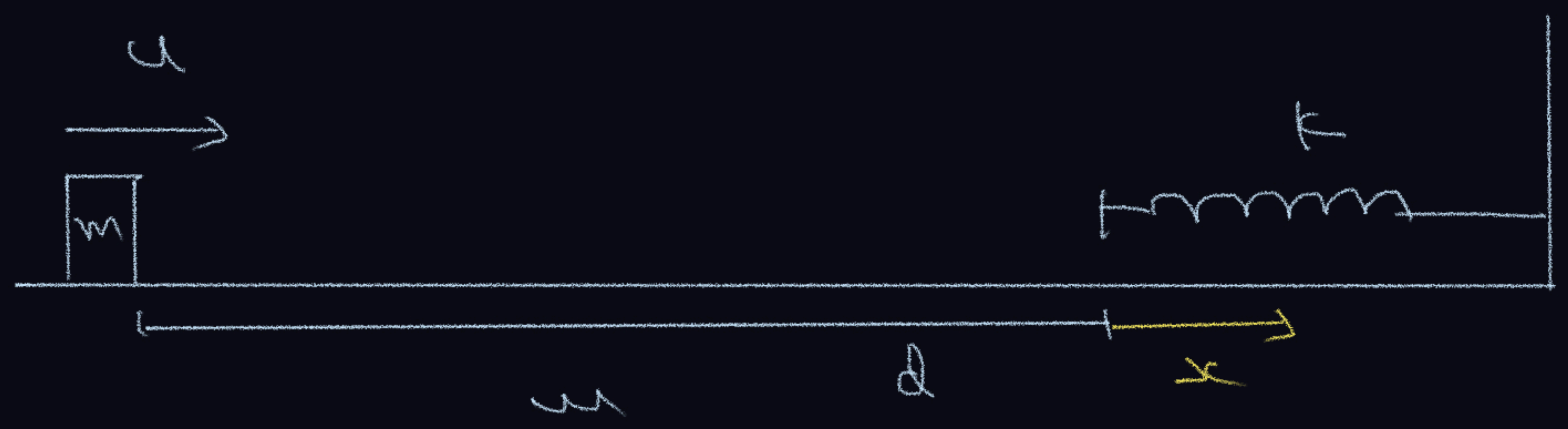




$$0 + \frac{1}{2} m u_0^2 + \frac{1}{2} k \left(\frac{mg}{k}\right)^2 = mgh + \frac{1}{2} k x^2 + \frac{1}{2} m 0^2$$

$h = x + x_0$

$$x = \sqrt{\frac{m}{k} u_0^2 + \left(\frac{mg}{k}\right)^2}$$



Find Max compression?

System: block + spring + gravitat

$$W_{ext} + W_{non} = \Delta K + \Delta PE$$

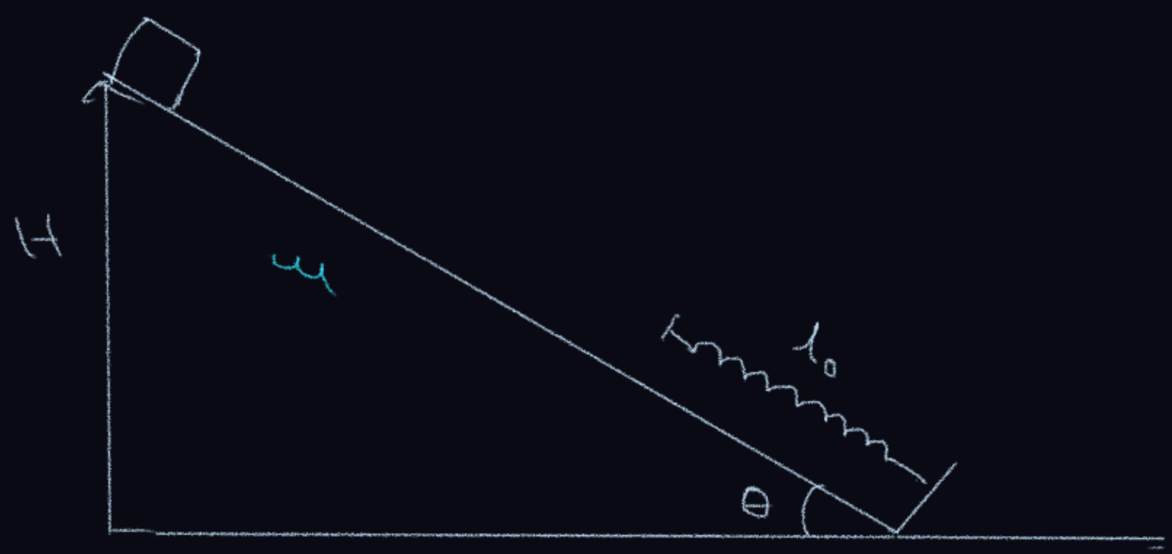
Normal, friction, spring, gravity

ext, Noncons, cons, cons

$$W_N + W_f = \Delta K + \Delta P$$

$$0 - \mu mg(d+x) = \left(0 - \tfrac{1}{2}mu^2\right) + \left(\tfrac{1}{2}kx^2 - 0\right)$$

$$\tfrac{1}{2}kx^2 - \mu mg x - \mu mg d + \tfrac{1}{2}mu^2 = 0$$

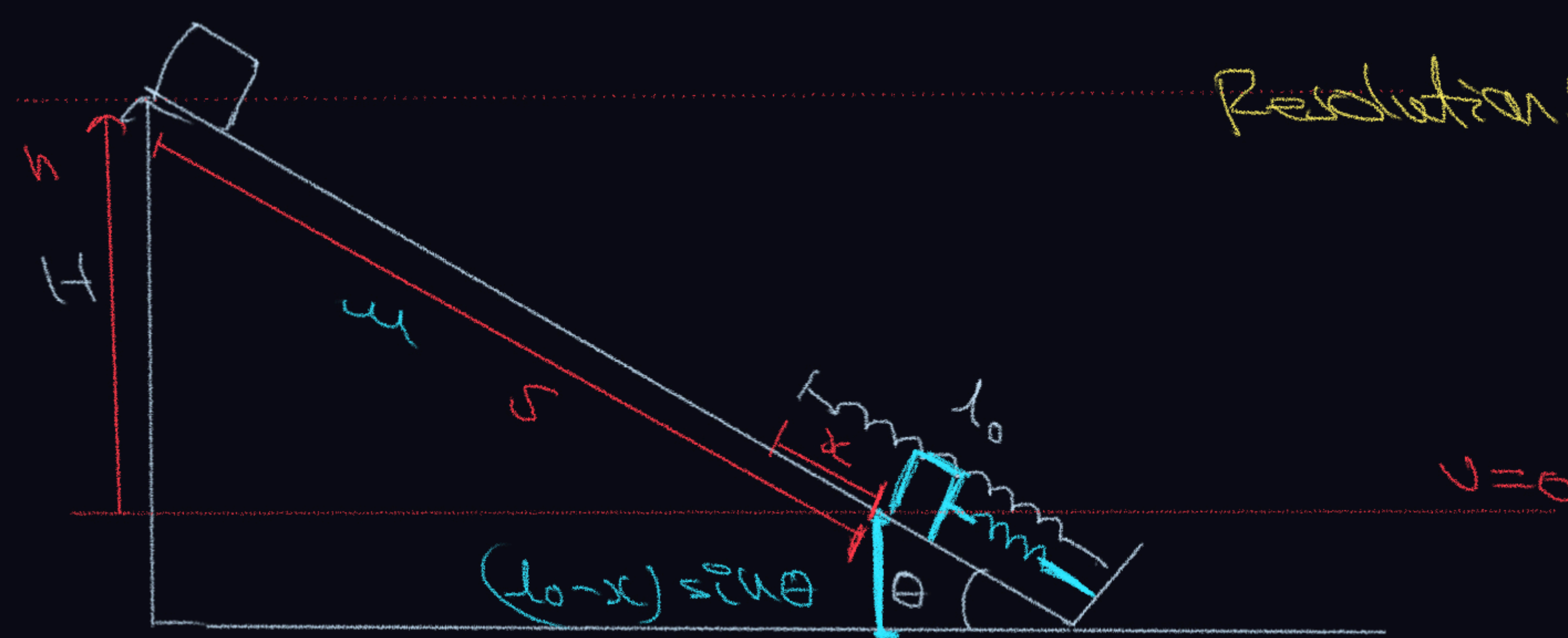


Find max compression in the spring?

$$W_N + W_f = (K_2 - K_1) + (P_2 - P_1) + (P_2 - P_1)$$

$$\boxed{0 - \mu mg\cos\theta(S) = (0-0) + (0 - mgh) + \left(\frac{1}{2}kx^2 - 0\right)}$$

$h = (x, H, \theta, l_0)$ $S = (x, H, \theta, l_0)$



Resolution:

$$h = H - (l_0 - x)\sin\theta$$

$$S = \frac{h}{\sin\theta} = \frac{H - (l_0 - x)\sin\theta}{\sin\theta}$$

Vertical Circle:

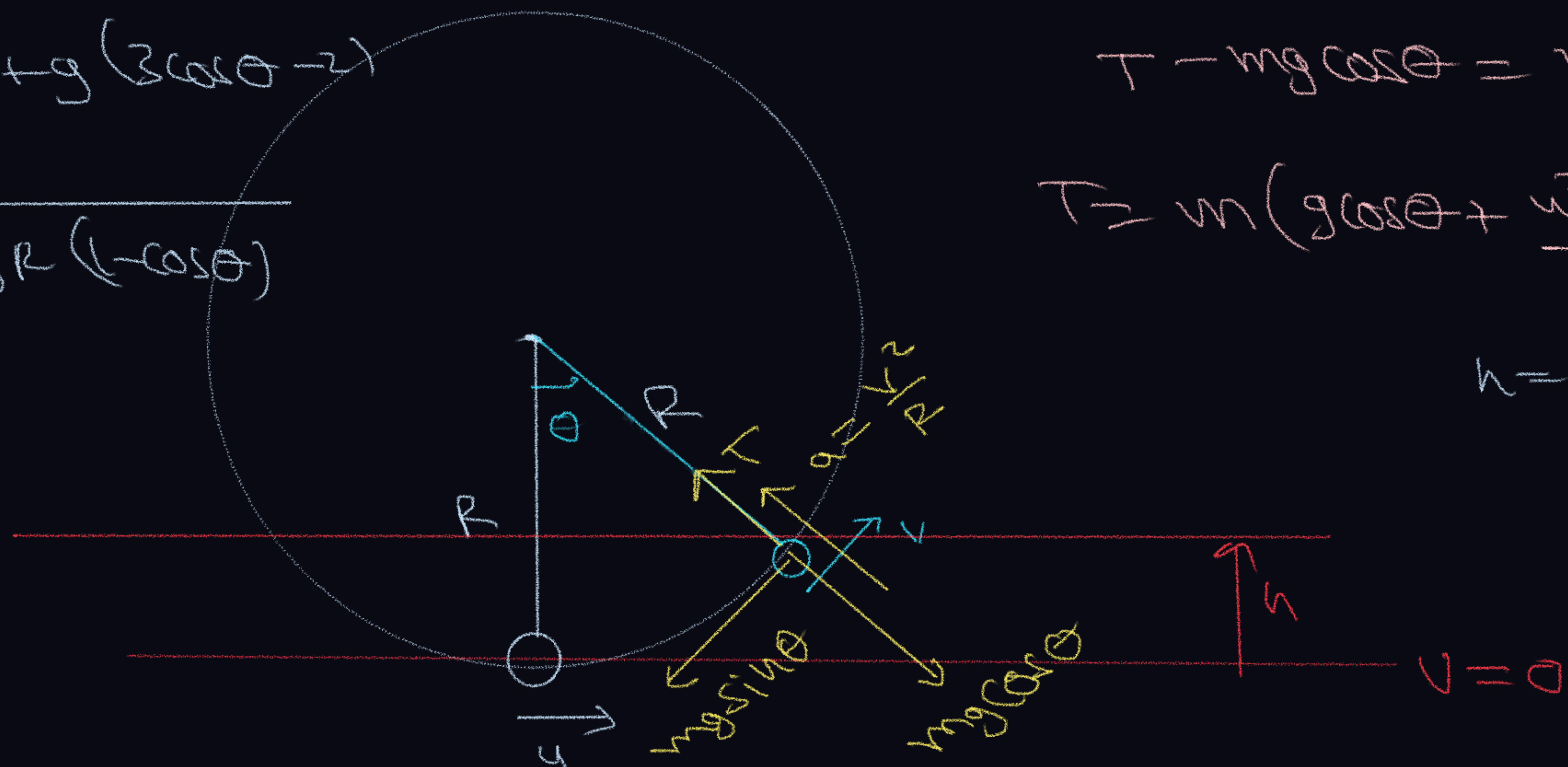
$$T = m\left(\frac{u^2}{R} + g(3\cos\theta - 2)\right)$$

$$V = \sqrt{u^2 - 2gR(1-\cos\theta)}$$

$$T - mg\cos\theta = \frac{mv^2}{R}$$

$$T = m\left(g\cos\theta + \frac{u^2 - 2gR(1-\cos\theta)}{R}\right)$$

$$h = R - R\cos\theta$$



$$T = m\left(\frac{u^2}{R} + g(3\cos\theta - 2)\right)$$

Energy theorem:-

$$W_T = \Delta K + \Delta P$$

$$0 = \left(\tfrac{1}{2}mv^2 - \tfrac{1}{2}mu^2\right) + \left(mgR(1-\cos\theta) - 0\right)$$

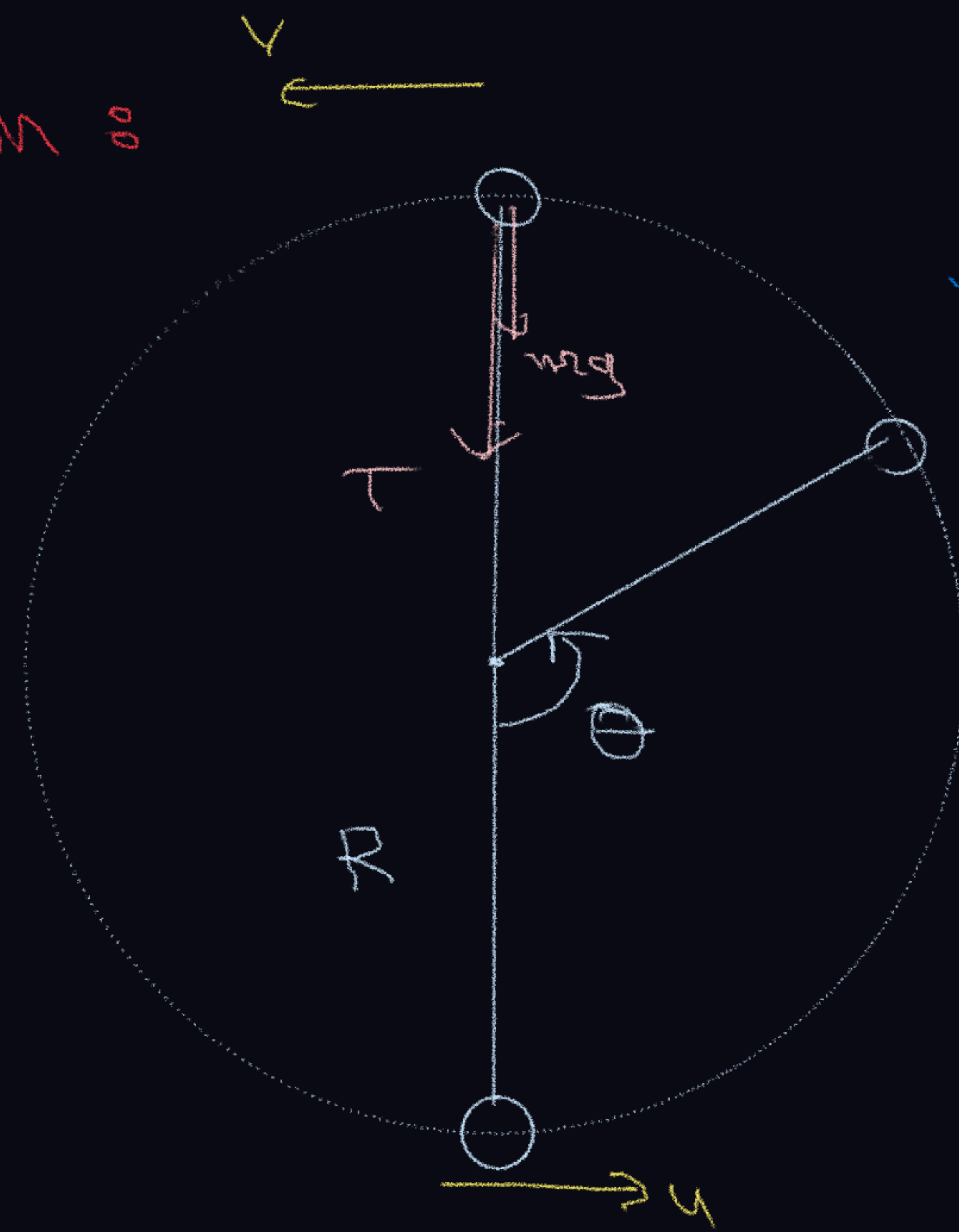
$$V = \sqrt{u^2 - 2gR(1-\cos\theta)}$$

Verticle circle condion :

$$T = m\left(\frac{v^2}{R} - g\right)$$

$$v^2 = u^2 - 4gR$$

$$\boxed{T_{top} = m\left(\frac{u^2}{R} - 5g\right)}$$



Verticle circle: $T \geq 0$

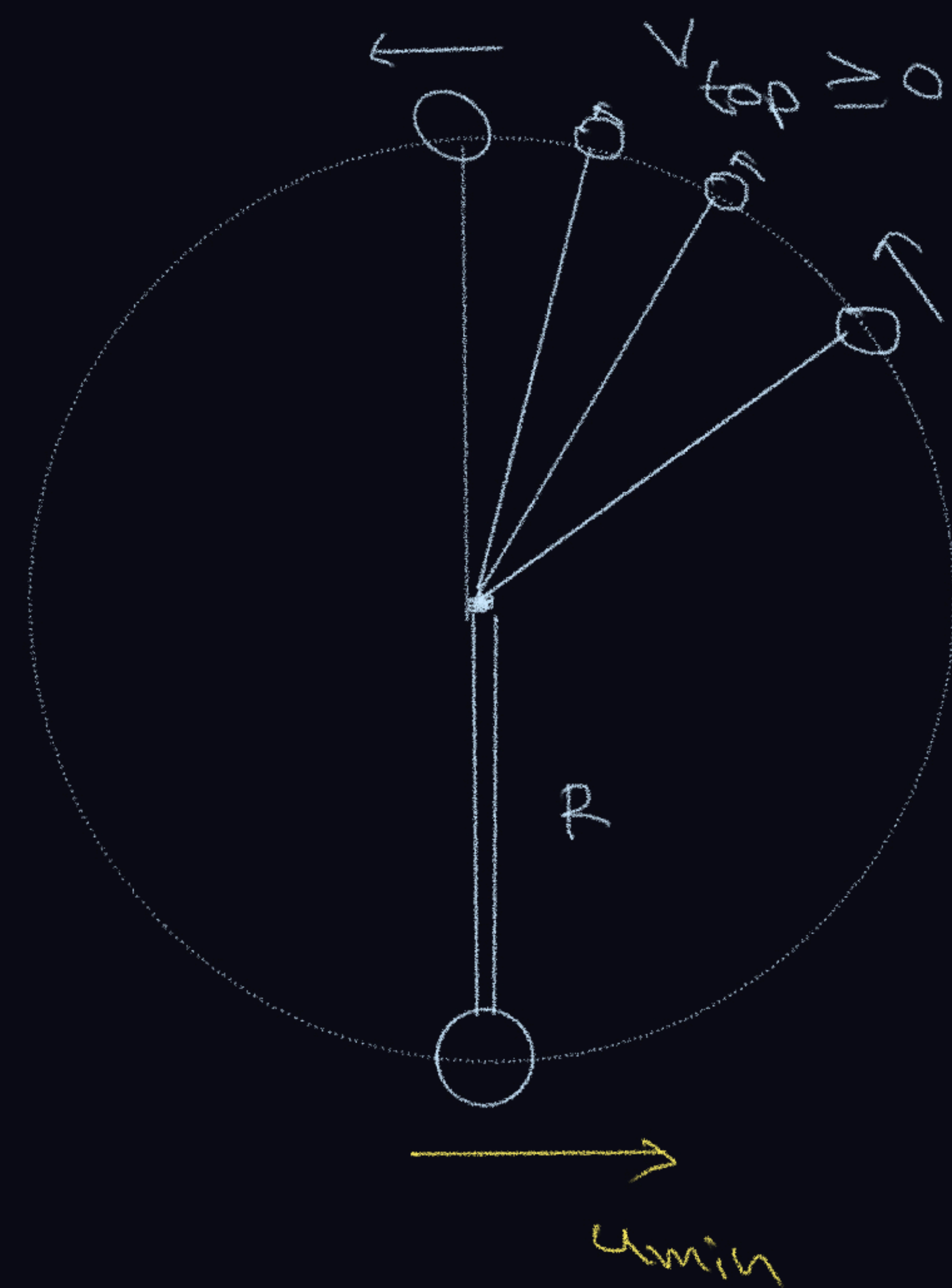
$$T = m\left(\frac{u^2}{R} + g(3\cos\theta - 2)\right)$$

$\theta \in (0, \pi)$ $\cos\theta \in (1, -1)$

when $\theta = \pi$ $T = T_{min}$

$$u_{min} = \sqrt{5gR}$$

Verticle circle with Rod:

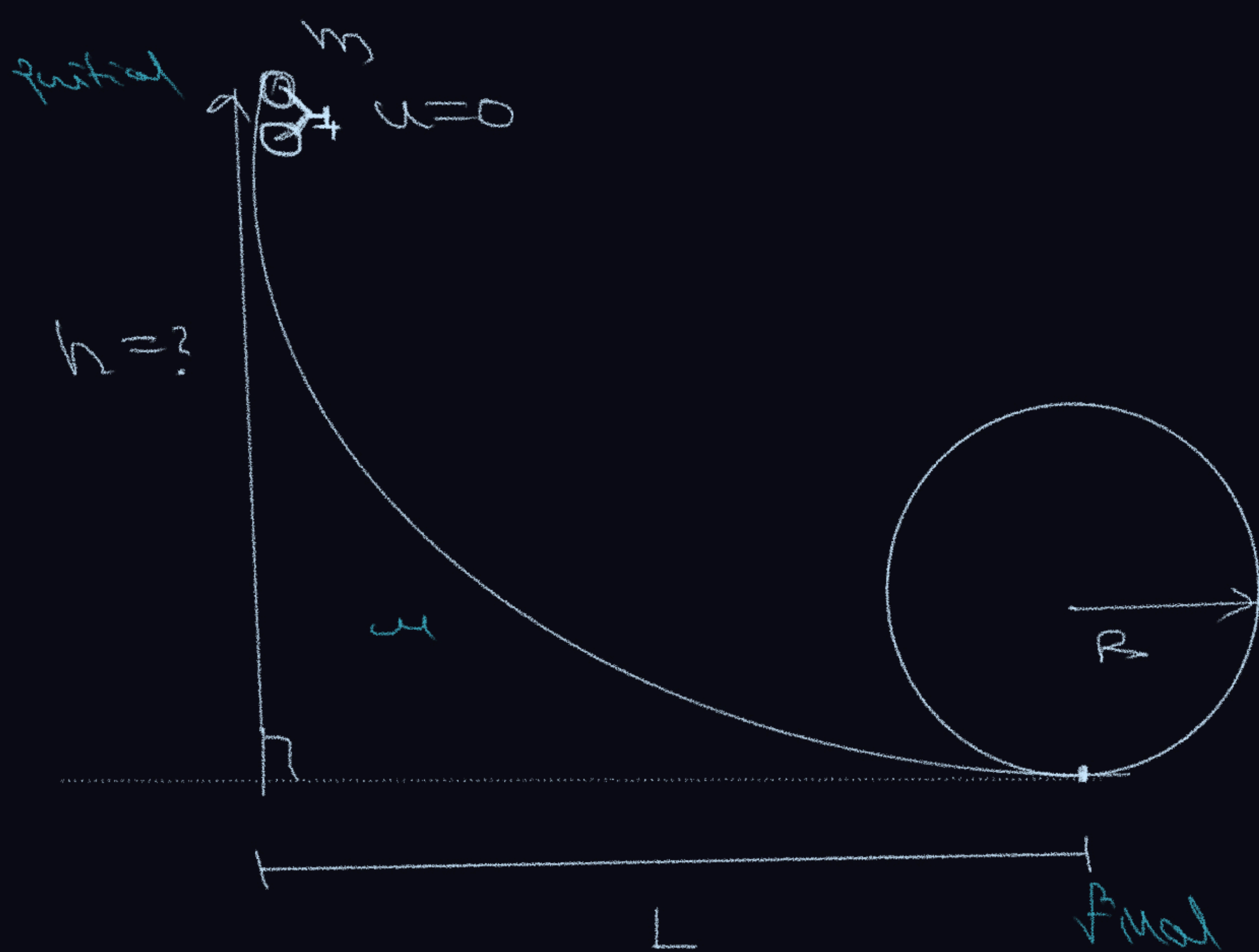


$$K_1 + P_1 = K_2 + P_2$$

$$\frac{1}{2}mu^2 + 0 = 0 + mg(2R)$$

$$u^2 = 4gR$$

$$u = \sqrt{4gR}$$



$$W_{ext} + W_{non} = \Delta K + \Delta P$$

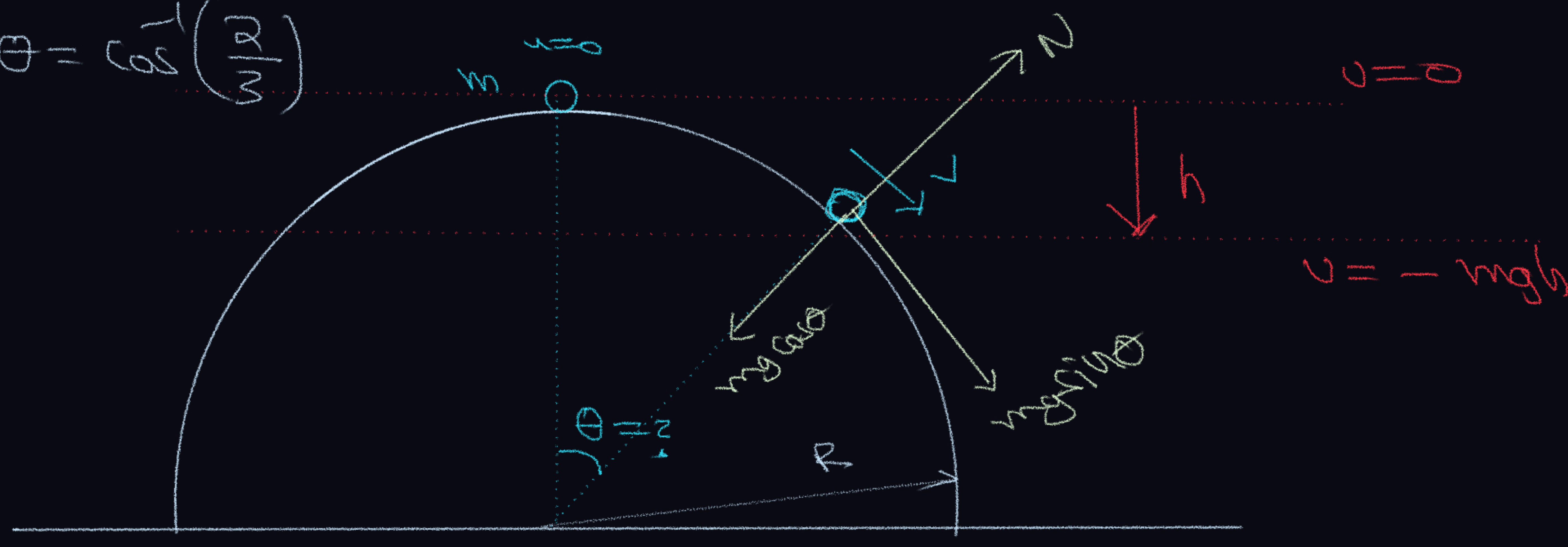
$$W_N + W_f = \Delta K + \Delta P$$

$$0 - \mu mgl = \Delta K + \Delta P$$

$$-\mu \cancel{m} gl = \frac{1}{2}\cancel{m}(5gR) - \cancel{m}gh$$

$$h = \frac{5R}{2} + \mu$$

$\theta = \cos^{-1}\left(\frac{2}{3}\right)$



$$K_1 + P_1 = K_2 + P_2$$

$$0 + 0 = \frac{1}{2}mv^2 - mgR(1-\cos\theta)$$

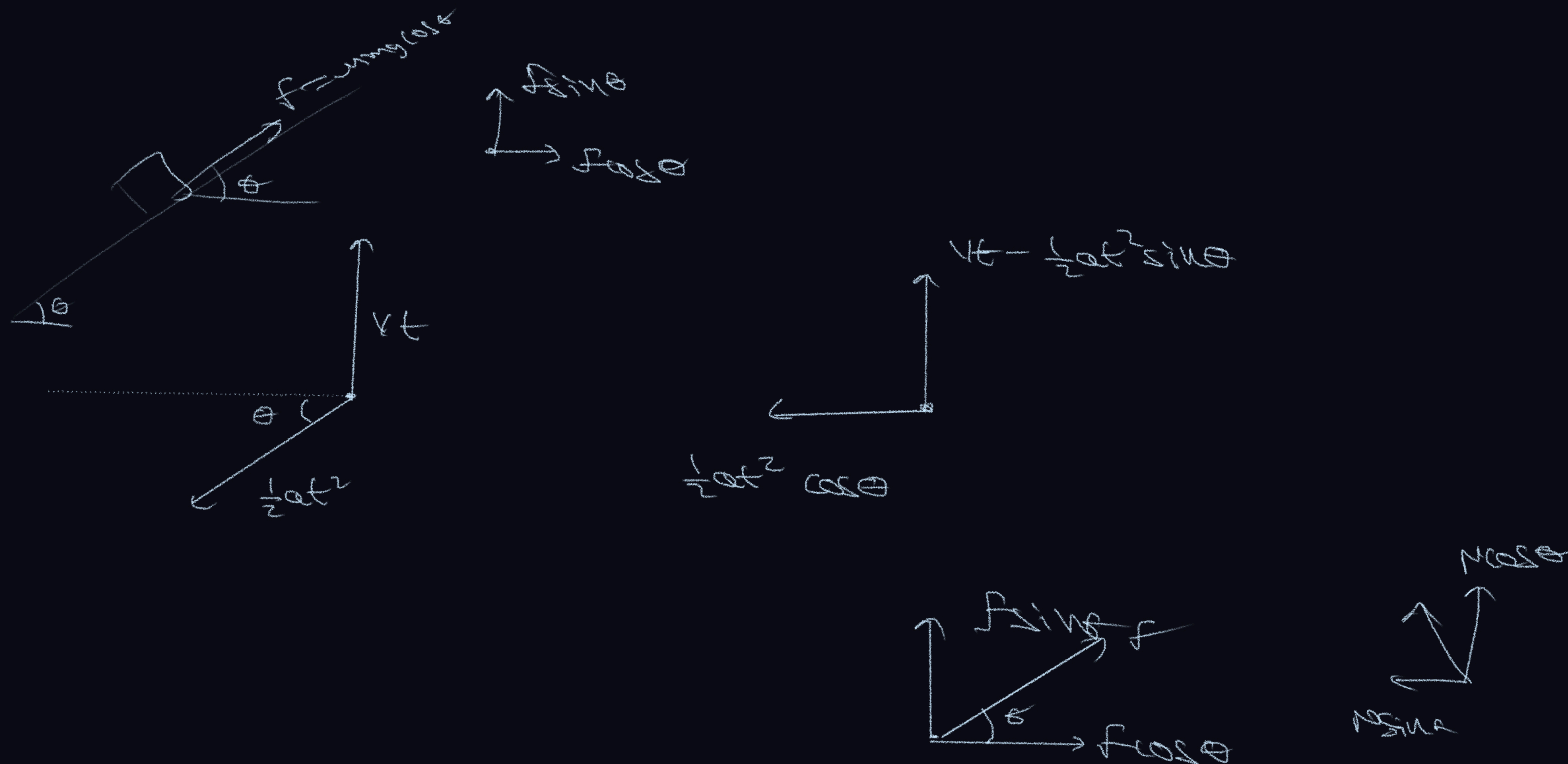
$$mg\cos\theta = m\frac{v^2}{R}$$

$$v^2 = gR\cos\theta$$

$$\frac{1}{2}mv^2 = mgR(1-\cos\theta) = \frac{1}{2}m(gR\cos\theta)$$

$$2\cancel{gR}(1-\cos\theta) = \cancel{gR}\cos\theta$$

$$2 = 3\cos\theta$$



$$W_{f_y} = \left(vt - \frac{1}{2}at^2\sin\theta\right)(f\sin\theta)$$

$$W_{f_x} = -f\cos\theta\left(\frac{1}{2}at^2\cos\theta\right)$$

$$F = F_x\hat{i} + F_y\hat{j}$$

$$s = x\hat{i} + y\hat{j}$$

$$xF_x + yF_y$$

$$W_x + W_y$$